1. $f(x,y) = x^2 e^{2y}$

Gradient is $\begin{matrix} 2xe^{2y} \\ 2x^2e^{2y} \end{matrix}$, $Df(2,0) = (4,8)$

Hessian is $\begin{matrix} 2e^{2y} & 4xe^{2y} \\ 4xe^{2y} & 4x^2e^{2y} \end{matrix} \qquad \begin{matrix} 2 & 8 \\ 8 & 16 \end{matrix}$

2.

$$H^* = \left\{ (r,\varphi) \in \mathbb{R}^2 \,\middle|\, 0 \le r \le 1, 0 \le \varphi \le \frac{\pi}{2} \right\}$$

$$\iint_H f(x,y)dxdy = \int_0^1 \left(\int_0^{\frac{\pi}{2}} \frac{r^2 \cos\varphi \sin\varphi}{\sqrt{r^2+1}} r d\varphi \right) dr =$$

$$\int_0^1 \frac{r^3}{r+1} dr \int_0^{\frac{\pi}{2}} \cos\varphi \sin\varphi d\varphi =$$

$$\int_0^1 \frac{r^3}{r+1} dr \left[\frac{\sin^2\varphi}{2} \right]_0^{\frac{\pi}{2}}$$

$$\frac{r^3}{r+1} = r^2 - r + 1 - \frac{1}{r+1}$$

$$\int_0^1 \frac{r^3}{r+1} dr = \left[\frac{r^3}{3} - \frac{r^2}{2} + r - \ln(r+1) \right]_0^1$$

$$\int_0^1 \left(\int_0^{\frac{\pi}{2}} \frac{r^2 \cos\varphi \sin\varphi}{\sqrt{r^2+1}} r d\varphi \right) dr = \frac{1}{4} \int_0^1 \frac{r^3}{r+1} dr = \frac{5}{12} - \frac{1}{2} \ln 2$$

3.

$$\begin{aligned} x'(t) &= \frac{1}{\sin\left(\frac{x(t)}{t}\right)} + \frac{x(t)}{t} \\ x(2) &= \frac{2\pi}{3} \end{aligned}$$

, Exact solution is: $\left\{ t \arccos\left(\ln 2 - \ln t + \frac{1}{2} \right) \right\}$

$$\begin{aligned} \frac{x(t)}{t} &= y(t) \\ y(t) + ty'(t) &= \frac{1}{\sin(y(t))} + y(t) \\ y'(t) &= \frac{1}{t \sin(y(t))} \end{aligned}$$

$$
\begin{aligned}
y'(t) &= \frac{1}{t\sin(y(t))} \\
y(2) &= \frac{\pi}{3}
\end{aligned}
$$

$$
\begin{aligned}
y'(t)\sin(y(t)) &= \frac{1}{t} \\
\cos(y(t)) &= \ln(t) + C
\end{aligned}
$$

4.

$$
\begin{aligned}
x'(t) + 2x(t) &= t \\
x(0) &= \frac{3}{4}
\end{aligned}
$$

, Exact solution is: $\left\{\frac{1}{2}t + e^{-2t} - \frac{1}{4}\right\}$