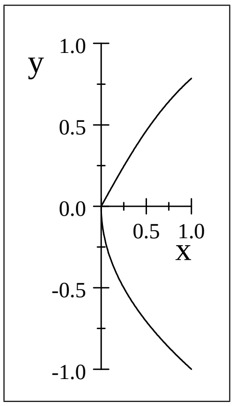
Anal2vizs-160606.tex = Analízis 2. vizsga, 2016.06.06. **MEGOLDÁSOK**

6) 

$$\mathbf{V}\acute{\imath} = \int\limits_{-1}^{0} \left(\int\limits_{y^2}^{1} 2x \; dx \right) dy + \int\limits_{0}^{1} \left(\int\limits_{\tan(y)}^{1} 2x \; dx \right) dy,$$

$$\mathbf{F\ddot{u}} = \int\limits_{0}^{1} \left(\int\limits_{-\sqrt{x}}^{\arctan(x)} 2x \; dy \right) dx = \int\limits_{0}^{1} 2x \cdot \left(\arctan(x) + \sqrt{x}\right) \; dx =$$

$$= \int\limits_{0}^{1} 2x \left(\arctan(x) + \sqrt{x}\right) \; dx = 2 \left[B + \frac{x^{5/2}}{5/2} \right]_0^1 =$$

$$= 2 \left[\frac{x^2}{2} \cdot \arctan(x) - \frac{x}{2} + \frac{1}{2} \arctan(x) + \frac{x^{5/2}}{5/2} \right]_0^1$$

mert $\quad B = \int x \cdot \arctan(x) \; dx = \frac{x^2}{2} \cdot \arctan(x) - \frac{1}{2} \int \frac{x^2}{1+x^2} dx =$

$$= A - \frac{1}{2} \int \frac{x^2+1-1}{1+x^2} dx = A - \frac{x}{2} + \frac{1}{2} \arctan(x) + C$$

7) $\quad x'(t) = \dfrac{x(t)}{\ln(x(t)) \cdot t^3} \;, \quad t = -1 \;, \; x = \frac{1}{e} \;,$

$$\iff \frac{x'(t) \cdot \ln(x(t))}{x(t)} = t^{-3} \iff \int \frac{\ln(x)}{x} dx = \int t^{-3} dt$$

$$\iff \frac{\ln^2(x)}{2} = \frac{t^{-2}}{-2} + C \iff \ln(x) = \pm \sqrt{C - \frac{1}{t^2}} \;,$$

K.É.P. $\implies -1 = -\sqrt{C - \dfrac{1}{(-1)^2}} \implies C = 2 \;,$

$$\implies x(t) = \exp\left(-\sqrt{2 - \frac{1}{t^2}} \right) = e^{-\sqrt{2-1/t^2}} \;.$$

8) $x''(t) - 4x'(t) + 4x(t) = \sin(t) \quad (*)$,

$\lambda^2 - 4\lambda + 4 = 0 \implies \lambda_{1,2} = 2 \implies x_{\text{hom}}(t) = c_1 e^{2t} + c_2 t e^{2t}$,

$x_{inh}^{pr}(t) = A\sin(t) + B\cos(t)$,

$x'(t) = A\cos(t) - B\sin(t)$,

$x''(t) = -A\sin(t) - B\cos(t) \qquad (= -x(t))$,

$(*) \implies$

$(-A\sin - B\cos) - 4\cdot(A\cos - B\sin) + 4\cdot(A\sin + B\cos) = \sin$,

$\sin\cdot(3A + 4B) + \cos\cdot(3B - 4A) = \sin$,

$$\implies \left.\begin{array}{c} 3A + 4B = 1 \\ 3B - 4A = 0 \end{array}\right\} \implies A = \frac{3}{25},\ B = \frac{4}{25}$$

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