Anal-2zh-2-170509.tex

A csoport

1) Feladat: $\lim\limits_{(x,y)\to(1,1)} \dfrac{\sin(x^2-y^2)}{x-y} = \dfrac{(x+y)\cdot\sin(x^2-y^2)}{(x+y)\cdot(x-y)} = (x+y)\cdot\dfrac{\sin(x^2-y^2)}{(x^2-y^2)}$

$= (x+y)\cdot\dfrac{\sin(t)}{(t)} \to (1+1)\cdot 1 = \mathbf{2}$, mert $t = x-y \to 0$ és $\lim\limits_{t\to 0}\dfrac{\sin(t)}{t} = 0$.

2) Feladat: Az $\vec{\mathbf{u}} = (u_1, u_2)$ irány szerinti derivált a $P(2,0)$ pontban, definíció szerint:

$$(D_{\mathbf{u}}f)(P) = \lim_{t\to 0}\frac{f(\ell(t)) - f(\ell(0))}{t-0} \tag{1}$$

ahol $\ell(t) = P + t\cdot\mathbf{u} = (2,0) + t\cdot(u_1,u_2) = (2+t\cdot u_1\ ,\ 0+t\cdot u_2)$, $u_1^2+u_2^2 = 1$ (és $t_0 = 0$).

(1) kiszámolása: mivel $x = 2+t\cdot u_1$ és $y = 0+t\cdot u_2$, ezért $(1) = \dfrac{\sqrt[3]{(2+t\cdot u_1-2)^2 - 4\cdot(0+t\cdot u_2)^2}}{t}$

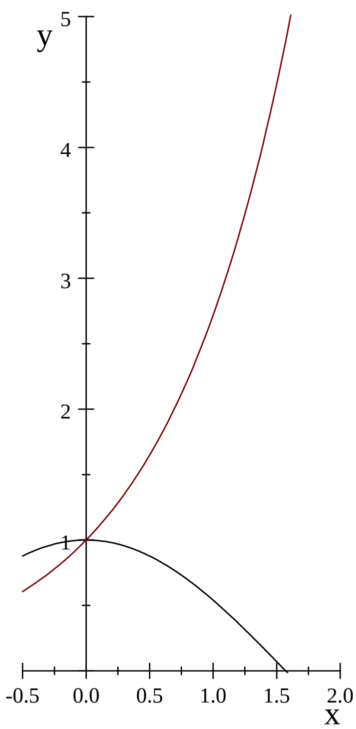
$= \dfrac{\sqrt[3]{t^2\cdot u_1^2 - 4\cdot t^2\cdot u_2^2}}{t} = \dfrac{\sqrt[3]{t^2}\cdot\sqrt[3]{u_1^2-4u_2^2}}{t} = \dfrac{\sqrt[3]{u_1^2-4u_2^2}}{\sqrt[3]{t}} = (*)$ hiszen $\frac{\sqrt[3]{t^2}}{t} = t^{2/3-1} = t^{-1/3} =$ $\frac{1}{\sqrt[3]{t}}$. Továbbá $\lim\limits_{t\to 0}(*) = \lim\limits_{t\to 0}\dfrac{\sqrt[3]{u_1^2-4u_2^2}}{\sqrt[3]{t}}$ csak akkor létezhet, ha $u_1^2 - 4u_2^2 = 0$, vagyis $u_1 = \pm 2u_2$. Tehát a keresett *irányvektorok* $\vec{\mathbf{u}}_1 = (2u_2, u_2) = u_2\cdot(2,1)$ és $\vec{\mathbf{u}}_2 = (-2u_2, u_2) = u_2\cdot(-2,1)$. Ha még az $u_1^2+u_2^2 = 1$ feltételt is teljesíteni akarjuk, akkor

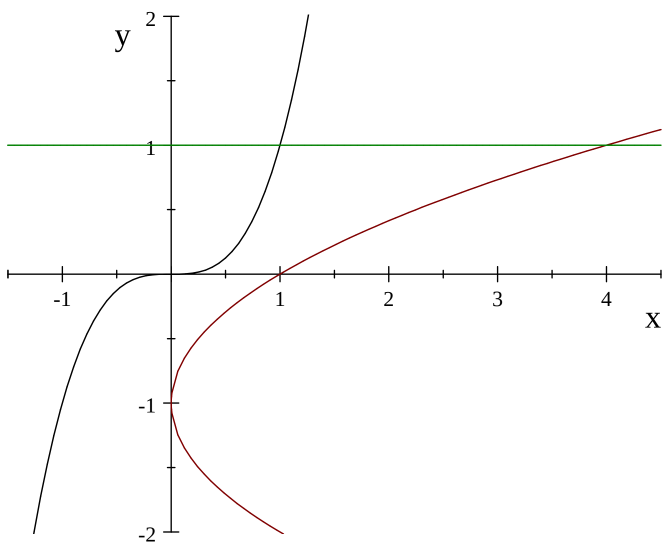
$\vec{\mathbf{u}}_1 = \pm\left(\frac{2}{\sqrt{2^2+1^2}}\ ,\ \frac{1}{\sqrt{2^2+1^2}}\right) = \pm\left(\frac{2}{\sqrt{5}}\ ,\ \frac{1}{\sqrt{5}}\right)$ és

$\vec{\mathbf{u}}_2 = \pm\left(\frac{-2}{\sqrt{(-2)^2+1^2}}\ ,\ \frac{1}{\sqrt{(-2)^2+1^2}}\right) = \pm\left(\frac{-2}{\sqrt{5}}\ ,\ \frac{1}{\sqrt{5}}\right)$.

6) Feladat: a B) csoport függvényében x és y -t kell felcserélni!

7) Feladat: $\cos(x)$ és e^x, $e^{\pi/2} \approx 4.8105$ **8) Feladat:** $x = \sqrt[3]{y}$ és $x = (y+1)^2$:



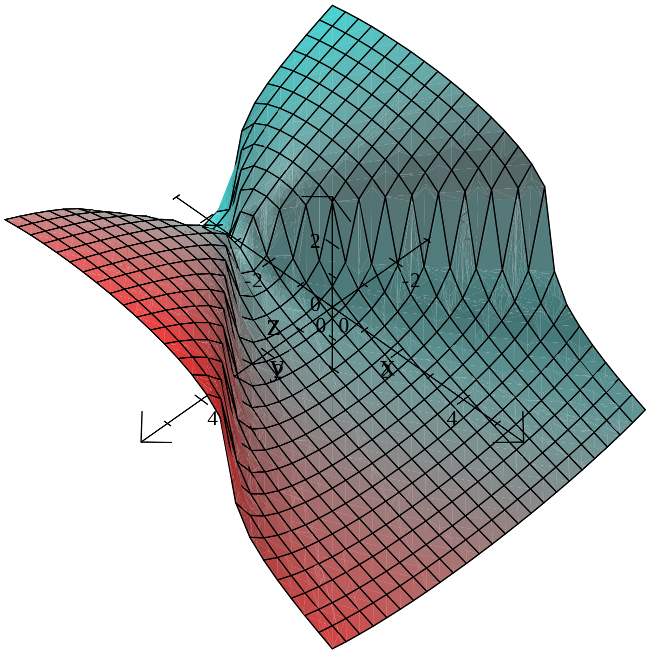


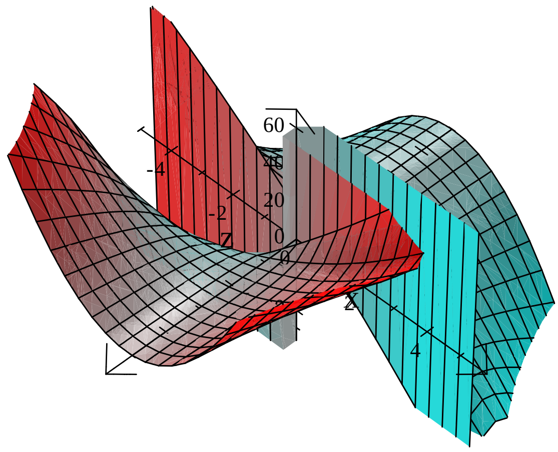
B csoport

1) Feladat: $\lim\limits_{(x,y)\to(2,2)} \dfrac{\sqrt{xy}-y}{x-y} = \dfrac{\sqrt{y}\left(\sqrt{x}-\sqrt{y}\right)}{\left(\sqrt{x}+\sqrt{y}\right)\left(\sqrt{x}-\sqrt{y}\right)} = \dfrac{\sqrt{y}}{\left(\sqrt{x}+\sqrt{y}\right)} \to \dfrac{\sqrt{2}}{2\sqrt{2}} = \dfrac{1}{2}$

2) Feladat: $\sqrt[3]{x^2-(y+3)^2}$

6) Feladat: $f(x,y) = y + \dfrac{cy}{x} + xy^2$, $x \neq 0$, pl. $c = 7$





6) Feladat: $\quad f(x,y) = y + \dfrac{cy}{x} + xy^2 \;, \quad x \neq 0 \;,$

$$D_1 f = \frac{df}{dx} = \frac{-cy}{x^2} + y^2 = 0 \iff y_1 = 0 \vee y = \frac{c}{x^2} \;,$$

$$D_2 f = \frac{df}{dy} = 1 + \frac{c}{x} + 2yx = 0 \;,$$

ha $\quad y_1 = 0 \implies D_2 f = 1 + \dfrac{c}{x} = 0 \implies x_1 = -c \implies \boxed{P_1(-c,0)}$,

ha $\quad y \neq 0 \implies D_2 f = 1 + \dfrac{c}{x} + 2\cdot\dfrac{c}{x^2}\cdot x = 1 + \dfrac{c}{x} + 2\cdot\dfrac{c}{x} = 1 + 3\dfrac{c}{x} = 0 \implies x_2 = -3c$

$\implies y_2 = \dfrac{c}{(-3c)^2} = \dfrac{1}{9c} \implies \boxed{P_2\left(-3c, \dfrac{1}{9c}\right)}$, ELL: OK

$D_{1,1} f = \dfrac{2cy}{x^3} \;, \quad D_{2,2} f = 2x \;, \quad D_{1,2} f = \dfrac{-c}{x^2} + 2y \;, \quad \boxed{\Delta = \dfrac{2cy}{x^3}\cdot 2x - \left(\dfrac{-c}{x^2} + 2y\right)^2}$,

$\Delta(P_1) = \dfrac{2c\cdot 0}{x^3}\cdot 2x - \left(\dfrac{-c}{x^2} + 2\cdot 0\right)^2 < 0 \implies$ nincs sz.é.

$\Delta(P_2) = \dfrac{2c\cdot\frac{1}{9c}}{(-3c)^3}\cdot 2\cdot(-3c) - \left(\dfrac{-c}{(-3c)^2} + 2\cdot\dfrac{1}{9c}\right)^2 = \dfrac{1}{27c^2} > 0 \implies$ van sz.é,

mégpedig $D_{1,1} f = \dfrac{2c\cdot\frac{1}{9c}}{(-3c)^3} = \dfrac{-2}{243c^3}$ max. vagy min. c előjelétől függően.

Megjegyzés: $y = 0$ esetén $f(x,y) = 0$.

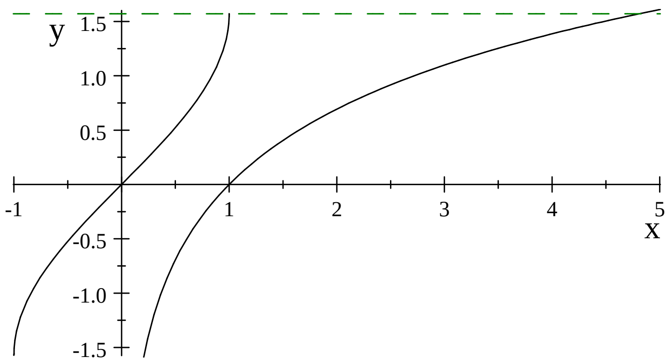
5) Feladat: $f(x,y) = g(\mathbf{u}(x,y)) \implies$

$$\operatorname{grad}(f) = [D_1 f\ ,\ D_2 f] = \operatorname{grad}(g)(\mathbf{u}) \cdot J(\mathbf{u}) = [D_1 g\ ,\ D_2 g] \cdot \begin{bmatrix} D_1 u_1 & D_2 u_1 \\ D_1 u_2 & D_2 u_2 \end{bmatrix} =$$

$$= [D_1 g \cdot D_1 u_1 + D_2 g \cdot D_1 u_2\ ,\ D_1 g \cdot D_2 u_1 + D_2 g \cdot D_2 u_2]$$

ahol $\mathbf{u}(x,y) = [u_1(x,y)\ ,\ u_2(x,y)]$,

7) Feladat: $\arcsin(x)$ és $\ln(x)$, $e^{\pi/2} \approx 4.8105$



8) Feladat: x^2 és x^3