

Gyakorló feladatok megoldásai - 7.

MA1112a – 2004/05

1.

$$\begin{array}{lll} \text{(a)} & 6 - 5i & \text{(b)} \quad -1 - 4i & \text{(c)} \quad -5 + i \\ \text{(d)} & 15 + 5i & \text{(e)} \quad -\frac{1}{2} - \frac{1}{2}i & \text{(f)} \quad -3 - i \end{array}$$

2. (a) $9(\cos \pi + i \sin \pi) = 9e^{\pi}$

(b) $5(\cos \frac{3}{2}\pi + i \sin \frac{3}{2}\pi) = 5e^{\frac{3}{2}\pi}$

(c) $3(\cos \frac{4}{3}\pi + i \sin \frac{4}{3}\pi) = 3e^{\frac{4}{3}\pi}$

(d) $4(\cos \frac{5}{6}\pi + i \sin \frac{5}{6}\pi) = 4e^{\frac{5}{6}\pi}$

(e) $4(\cos \frac{5}{3}\pi + i \sin \frac{5}{3}\pi) = 4e^{\frac{5}{3}\pi}$

(f) $3\sqrt{2}(\cos \frac{3}{4}\pi + i \sin \frac{3}{4}\pi) = 3\sqrt{2}e^{\frac{3}{4}\pi}$

(g) $2\sqrt{5}(\cos(\pi - 2) + i \sin(\pi - 2)) \approx 4.4721(\cos 2.0344 + i \sin 2.0344) = 4.4721e^{2.034}$

(h) $\sqrt{10}(\cos(2\pi - 3) + i \sin(2\pi - 3)) \approx 3.1623(\cos 5.0341 + i \sin 5.0341) = 3.1623e^{5.0341}$

3.

$$\text{(a)} \quad -128 + 128\sqrt{3}i \quad \text{(b)} \quad 2^{55} + 2^{55}i \quad \text{(c)} \quad 12\sqrt{3} - 12i$$

$$\text{(d)} \quad -\frac{\sqrt{3}}{2} - \frac{1}{2}i \quad \text{(e)} \quad \frac{3}{8} + \frac{3\sqrt{3}}{8}i \quad \text{(f)} \quad -\frac{\sqrt{3}}{2} + \frac{1}{2}i$$

4. (a) $-2, \quad 1 - \sqrt{3}i, \quad 1 + \sqrt{3}i$

(b) $\frac{\sqrt{3}}{2} + \frac{1}{2}i, \quad -\frac{\sqrt{3}}{2} + \frac{1}{2}i, \quad -i$

(c) $\sqrt[3]{18}(\cos \frac{7}{12}\pi + i \sin \frac{7}{12}\pi) = -0.41899 + 1.56371i$

$\sqrt[3]{18}(\cos \frac{15}{12}\pi + i \sin \frac{15}{12}\pi) = -1.1447 - 1.1447i$

$\sqrt[3]{18}(\cos \frac{23}{12}\pi + i \sin \frac{23}{12}\pi) = 1.5637 - 0.4190i$

(d) $3i, \quad -\frac{3\sqrt{3}}{2} - \frac{3}{2}i, \quad \frac{3\sqrt{3}}{2} - \frac{3}{2}i$

(e) $1, \quad \frac{1}{2} + \frac{\sqrt{3}}{2}i, \quad -\frac{1}{2} + \frac{\sqrt{3}}{2}i, \quad -1, \quad -\frac{1}{2} - \frac{\sqrt{3}}{2}i, \quad \frac{1}{2} - \frac{\sqrt{3}}{2}i$

(f) $2(\cos \frac{5}{15}\pi + i \sin \frac{5}{15}\pi) \approx 1 + 1.7321i$

$2(\cos \frac{11}{15}\pi + i \sin \frac{11}{15}\pi) \approx -1.3383 + 1.4863i$

$2(\cos \frac{17}{15}\pi + i \sin \frac{17}{15}\pi) \approx -1.8271 - 0.8135i$

$2(\cos \frac{23}{15}\pi + i \sin \frac{23}{15}\pi) \approx 0.2091 - 1.9890i$

$2(\cos \frac{29}{15}\pi + i \sin \frac{29}{15}\pi) \approx 1.9563 - 0.4158i$

5. (a) $0, 1, -\frac{1}{2} + \frac{\sqrt{3}}{2}i, -\frac{1}{2} - \frac{\sqrt{3}}{2}i$
(b) $\frac{1}{2}, 1 + \frac{\sqrt{2}}{2}i, 1 - \frac{\sqrt{2}}{2}i$
(c) $\sqrt{3} + i, -\sqrt{3} + i, -2i$
(d) $1 + i, 1 - i, -1 + i, -1 - i$
(e) $-\frac{1}{2} + \frac{\sqrt{3}}{2}i, -\frac{1}{2} - \frac{\sqrt{3}}{2}i, \frac{1}{2} + \frac{\sqrt{3}}{2}i, \frac{1}{2} - \frac{\sqrt{3}}{2}i$
(f) $-\sqrt[3]{5}, \sqrt[3]{5}(\frac{1}{2} + \frac{\sqrt{3}}{2}i), \sqrt[3]{5}(\frac{1}{2} - \frac{\sqrt{3}}{2}i), 1, -\frac{1}{2} + \frac{\sqrt{3}}{2}i, -\frac{1}{2} - \frac{\sqrt{3}}{2}i$
6. (a) $\cos^3 \varphi - 3 \cos \varphi \sin^2 \varphi$
(b) $3 \sin \varphi \cos^2 \varphi - \sin^3 \varphi$
(c) $\cos^4 \varphi - 6 \cos^2 \varphi \sin^2 \varphi + \sin^4 \varphi$