

1) Give $f \circ g$ where

$$\begin{aligned} f(x) &:= \frac{\ln(x+1)}{x-1}, & x \in (1, 5), \\ g(x) &:= \sqrt{x-2} - 3, & x \in [4, 32]. \end{aligned} \quad (10p)$$

2) Give the inverse of the function $h(x) = (2 - \sqrt{x-1})^2$ if it does exist. (10p)

3) Calculate the following limits:

$$\lim_{n \rightarrow \infty} \frac{3n^2 - 43n + 597}{372n - 1024}, \quad \lim_{n \rightarrow \infty} \sqrt[3]{37n - 79 \arctg(n)}, \quad \lim_{n \rightarrow \infty} \frac{1.034^n}{n^{875}}. \quad (3 \times 5p)$$

4) Calculate the following limits:

$$\lim_{x \rightarrow \infty} \frac{\sqrt{x} + \sqrt[3]{x}}{2\sqrt{x}}, \quad \lim_{x \rightarrow 0} \frac{\sin(x^4)}{x^2}, \quad \lim_{x \rightarrow -3} \frac{x^2 + 2x - 3}{x^2 - 2x - 15}. \quad (3 \times 5p)$$

5) Calculate the (formal) derivative of the function:

$$k(x) := \arctg(\ln^5(x)) + 3 \frac{\sin(5x)}{\sqrt{x^4 - 7x}} + \exp(x^2). \quad (20p)$$

6) Determine the tangent line of the function $\ell(x) := 3 \frac{\sin(5x)}{\sqrt{x^4 - 7x}}$ at the point $x_0 := 2.1$. (10p)

7) Give the exact definition of the *finite (value) limit* of a function at a *finite place*. (10p)

8) Give the connection between the monotonicity of a function and the sign of its derivative (four theorems). (10p)

Total: 100p