Deriváló

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Derivalo-mego-211226.tex, 2022.01.19.

http://math.uni-pannon.hu/~szalkai/Derivalo-mego-211226.pdf

Számoljuk ki az alábbi függvények deriváltfüggvényeit ("formális deriválás")! Ügyeljünk a zárójelekre!

1.) $(x^2+3x-1)\left(x+\sqrt{2}\right)$, $(x-1)^2(x^2+2)$,

2.) $\dfrac{3x^2-2x+1}{\sqrt[3]{x}}$, $\dfrac{(x+1)^2}{x^2+1}$, $\dfrac{\sqrt{x}-x^3}{\sqrt[3]{x}}$,

3. !) $\left(\dfrac{x+1}{x-1}\right)^2$,

4. !!!) $\dfrac{2}{3+8x^5}$, $\dfrac{1}{\tan^2 x}$,

5. !) $\dfrac{1}{\sqrt{x}}-3\sqrt{x^2+5x}$, $\sqrt[3]{1+\sqrt[3]{2+5x}}$,

6.) $(3x^2+5x)\cos 3x$, $(2+x^2)\sin 2x$,

7. !!!) $e^{\sqrt{1+x}}$, 5^{1-x^2} , $3^{1/x}$,

8.) $\dfrac{1+\cos x}{1-\cos x}$, $\dfrac{\cos^2 x}{\cos x^2}$,

9.) $\sqrt{\sin x}$, $\sin\sqrt{x}$, $\sqrt[7]{chx}$,

10.) $\arcsin\dfrac{1}{x}$, $\arccos\left(1-x^2\right)$, $\arcsin\left(2x\sqrt{1-x^2}\right)$,

11. !!) $\ln\dfrac{x}{2+3x}$, $\ln\tan x$,

12. !!!) $\ln\sqrt{\dfrac{5x}{2+3x}}$, $\log_2\sqrt{\dfrac{5x}{2+3x}}$,

13.) $\dfrac{x}{\sin x+\cos x}$,

14. !!!) $(3x^2-2)\,e^2$,

15.) $(3x^2-2)\,e^{2x}$,

16.) $e^{2x}\cos 3x$,

17. !!!) $\tan\dfrac{1}{x^2}$, $x^2\tan 2x$,

18. *) $\dfrac{1}{chx}$, x^x , $x^{1/x}$,

19.) $\log_3(1-x^2)$,

20.) $\ln\dfrac{x^2-1}{x^2+1}$,

21.) $\sqrt{\dfrac{1+\cos x}{2}}$,

22.) $x^3(1-2x)(3x^2+4x)$,

23.) $\dfrac{e^{2\cos x}}{\sin x}$, $\dfrac{\sin(\cos(x^2))}{\exp_4\left(tg\left(\dfrac{1}{x^2}\right)\right)}$,

24.) $\dfrac{e^x}{x}$, e^{-x^2} , $\dfrac{1}{1+x^2}$, $\sqrt[5]{2^x}$,

25.) $2^x\left(1-\dfrac{1}{x}\right)$, $\sqrt[3]{1-x^3}$, $\log_x(100)$,

Gyakorlásra:

1.) $(1+x^2)^{1/\ln x}$,

2.) xe^{2/x^2} , $x^2e^{1/x}$, $\dfrac{e^{1/x}}{x-2}$, $\dfrac{x}{\sqrt[3]{x+2}}$,

3.) $2^{\left(1-x^2+\cos(x)\right)}\cdot\ln(x^2+x)$, $\log_5\left(\dfrac{x^2-2x}{\sqrt[3]{x}}\right)(x^3-12x^2)^2$,

4.) $\dfrac{x^7+4^7+4^x}{\log_3(2x)}$, $\dfrac{\cos(3x^2-x)}{tg(3^x)}$, $ctg(3x)\,(x^5-32x)^4$,

5.) $\dfrac{e^{2x-4x^3}}{\sqrt[4]{\cos(x)}}$, $\dfrac{(x^7+4x)^{2000}}{e^{x^2}}$, $\dfrac{\cos(2x^2+3x)}{2^{2x}}$,

6.) $\dfrac{x^{17}-8x^4+5x}{tg\left(x\right)}$, $\dfrac{\log_2\left(1-x\right)}{\sin\left(1+x\right)}$, $\left(10x^3-5x^2+1.7\right)4^x$,

7.) $\dfrac{\sin\left(4x\right)}{\sqrt[3]{x}}$, $\sin\left(x\right)\left(3^x-ctg\left(x\right)\right)$, $\log_7\left(x^3-\sqrt{x}\right)$,

8.) $\dfrac{\log_2\left(3x\right)}{7^{5x}+\sin\left(-x\right)}$, $\left(x^8-5\sqrt{x}\right)5^x$,

9.) $\dfrac{arctg\left(-x\right)}{\sqrt{x^2+1}}+\ln\left(x\right)3^{x^2}-\cos^4\left(tg\left(\frac{1}{x}\right)\right)$,

10.) $x^2\arcsin\left(2x\right)+\ln^3\left(\sqrt{x^2+5x}\right)-\dfrac{2^x}{\sin\left(x^3\right)}$,

11.) $2^{3x}\ln\left(x\right)-\dfrac{\tan\left(\sqrt{x}\right)}{\left(x^2+1\right)^4}+2\arcsin^5\left(1-3x^3\right)$,

12.) $\dfrac{\sqrt{-x}}{\tan\left(x\right)}+2\arcsin\left(2^{1-x^2}\right)-3\ln\left(x^4+4^{x+1}+1\right)\sin\left(x\right)$,

13.) $\arctan^2\left(\ln x\right)-2\sqrt{x}3^{x^3}+\dfrac{\cot\left(x\right)}{x^3}-x^{1/x}$,

14.) $e^{tg(x)}x^3+\sqrt[3]{\arcsin\ln x}-2\dfrac{\left(x^2-5x\right)^6}{\cos\left(4x\right)}$,

15.) $\dfrac{3\sin\left(6x\right)}{\sqrt[3]{x^4+2x}}+e^{x^2}\cot\left(-x\right)$,

16.) $\sqrt[3]{\cos x^3}+\sqrt{\cos^3 x}+2\sin\left(2x\right)\ln\left(1+\frac{1}{x}\right)-\dfrac{arctg\left(x^3\right)}{3^x}$,

17.) $\tan\left(3x\right)\sqrt[4]{\cos x+6x+3}+\dfrac{3\ln 3x}{\left(3x+3\right)^3}$,

18.) . . .

Megoldások

1)a) $\left[\left(x^2+3x-1\right)\left(x+\sqrt{2}\right)\right]'=\left(x^2+3x-1\right)'\left(x+\sqrt{2}\right)+\left(x^2+3x-1\right)\left(x+\sqrt{2}\right)'=$

$=(2x+3)\left(x+\sqrt{2}\right)+\left(x^2+3x-1\right)\cdot 1$,

b) $\left[(x-1)^2\left(x^2+2\right)\right]'=\left[(x-1)^2\right]'\left(x^2+2\right)+(x-1)^2\left(x^2+2\right)'=$

$=2\cdot(x-1)^{2-1}\cdot 1\cdot\left(x^2+2\right)+(x-1)^2\cdot(2x+0)$

2)a) $\left(\dfrac{3x^2-2x+1}{\sqrt[3]{x}}\right)'=\dfrac{\left(3x^2-2x+1\right)'\cdot\sqrt[3]{x}-\left(3x^2-2x+1\right)\cdot\left(x^{1/3}\right)'}{\left(\sqrt[3]{x}\right)^2}=$

$=\dfrac{\left(3\cdot 2\cdot x^{2-1}-2+0\right)\cdot\sqrt[3]{x}-\left(3x^2-2x+1\right)\cdot\left(\frac{1}{3}x^{1/3-1}\right)}{\left(\sqrt[3]{x}\right)^2}$

b) $\left(\dfrac{(x+1)^2}{x^2+1}\right)'=\dfrac{\left[(x+1)^2\right]'\cdot\left(x^2+1\right)-(x+1)^2\cdot\left(x^2+1\right)'}{\left(x^2+1\right)^2}=$

$=\dfrac{\left[2\cdot(x+1)^{2-1}\cdot 1\right]\cdot\left(x^2+1\right)-(x+1)^2\cdot\left(2x^{2-1}+0\right)}{\left(x^2+1\right)^2}$,

c) $\left(\dfrac{\sqrt{x}-x^3}{\sqrt[3]{x}}\right)'=\dfrac{\left(x^{1/2}-x^3\right)'\cdot\sqrt[3]{x}-\left(\sqrt{x}-x^3\right)\cdot\left(x^{1/3}\right)'}{\left(\sqrt[3]{x}\right)^2}=$

$=\dfrac{\left(\frac{1}{2}x^{\frac{1}{2}-1}-3x^{3-1}\right)\cdot\sqrt[3]{x}-\left(\sqrt{x}-x^3\right)\cdot\left(\frac{1}{3}x^{1/3-1}\right)}{\left(\sqrt[3]{x}\right)^2}$,

3) $\left[\left(\dfrac{x+1}{x-1}\right)^2\right]'=2\cdot\left(\dfrac{x+1}{x-1}\right)^{2-1}\cdot\left(\dfrac{x+1}{x-1}\right)'=$

$=2\cdot\left(\dfrac{x+1}{x-1}\right)^{2-1}\cdot\dfrac{(x+1)'\cdot(x-1)-(x+1)\cdot(x-1)'}{(x-1)^2}=2\cdot\left(\dfrac{x+1}{x-1}\right)^{2-1}\cdot\dfrac{1\cdot(x-1)-(x+1)\cdot 1}{(x-1)^2}$

4)a) $\left(\dfrac{2}{3+8x^5}\right)'=\left(2\cdot\left(3+8x^5\right)^{-1}\right)'=2\cdot(-1)\cdot\left(3+8x^5\right)^{-1-1}\cdot\left(0+8\cdot 5x^{5-1}\right)$,

b) $\left(\dfrac{1}{\tan^2 x}\right)'=\left(\left(\tan^2(x)\right)^{-1}\right)'=\left((\tan(x))^{-2}\right)'=-2\cdot(\tan(x))^{-2-1}\cdot\tan'(x)=$

$=-2\cdot(\tan(x))^{-2-1}\cdot\dfrac{1}{\cos^2(x)}$,

5)a) $\left(\dfrac{1}{\sqrt{x}}-3\sqrt{x^2+5x}\right)'=\left(x^{-\frac{1}{2}}\right)'-3\left(\left(x^2+5x\right)^{\frac{1}{2}}\right)'=$

$=\dfrac{-1}{2}\cdot x^{-\frac{1}{2}}-3\cdot\dfrac{1}{2}\cdot\left(x^2+5x\right)^{\frac{1}{2}-1}\cdot(2x+5)$

b) $\left(\sqrt[3]{1+\sqrt[3]{2+5x}}\right)'=\left(\left(1+(2+5x)^{\frac{1}{3}}\right)^{\frac{1}{3}}\right)'=$

$=\dfrac{1}{3}\cdot\left(1+(2+5x)^{\frac{1}{3}}\right)^{\frac{1}{3}-1}\cdot\left(0+\dfrac{1}{3}\cdot(2+5x)^{\frac{1}{3}-1}\cdot(0+5)\right)$,

6)a) $\left[(3x^2+5x)\cdot\cos(3x)\right]' = (3x^2+5x)'\cdot\cos(3x) + (3x^2+5x)\cdot(\cos(3x))' =$

$= (3\cdot 2x^{2-1}+5)\cdot\cos(3x) + (3x^2+5x)\cdot(-\sin(3x))\cdot 3$,

b) $\left[(2+x^2)\cdot\sin(2x)\right]' = (2+x^2)'\cdot\sin(2x) + (2+x^2)\cdot(\sin(2x))' =$

$= (0+2x^{2-1})\cdot\sin(2x) + (2+x^2)\cdot\cos(2x)\cdot 2$,

7)a) $\left(e^{\sqrt{1+x}}\right)' = \left(\exp\left(\sqrt{1+x}\right)\right)' = \exp'\left(\sqrt{1+x}\right)\cdot\left(\sqrt{1+x}\right)' = e^{\sqrt{1+x}}\cdot\left((1+x)^{\frac{1}{2}}\right)' =$

$= e^{\sqrt{1+x}}\cdot\frac{1}{2}(1+x)^{\frac{1}{2}-1}\cdot 1$

b) $\left(5^{1-x^2}\right)' = \left(\exp_5(1-x^2)\right)' = \exp_5'(1-x^2)\cdot(1-x^2)' = 5^{1-x^2}\cdot\ln(5)\cdot(1-2x)$,

c) $\left(3^{1/x}\right)' = \left(\exp_3\left(\frac{1}{x}\right)\right)' = \exp_3'\left(\frac{1}{x}\right)\cdot(x^{-1})' = 3^{1/x}\cdot\ln(3)\cdot(-1)\cdot 2\cdot x^{-1-1}$

8)a) $\left(\frac{1+\cos x}{1-\cos x}\right)' = \frac{(1+\cos x)'\cdot(1-\cos x)-(1+\cos x)\cdot(1-\cos x)'}{(1-\cos x)^2} =$

$= \frac{(0-\sin x)\cdot(1-\cos x)-(1+\cos x)\cdot(0+\sin x)}{(1-\cos x)^2}$,

b) $\left(\frac{\cos^2 x}{\cos x^2}\right)' = \frac{\left([\cos(x)]^2\right)'\cdot(\cos(x^2)) - \left([\cos(x)]^2\right)\cdot(\cos(x^2))'}{(\cos(x^2))^2} =$

$= \frac{2\cdot[\cos(x)]^{2-1}(-\sin x)\cdot(\cos(x^2)) - \left([\cos(x)]^2\right)\cdot(-\sin(x^2)\cdot 2x)}{(\cos(x^2))^2}$,

9)a) $\left(\sqrt{\sin x}\right)' = \left((\sin(x))^{\frac{1}{2}}\right)' = \frac{1}{2}\cdot(\sin(x))^{\frac{1}{2}-1}\cdot\sin'(x) = \frac{1}{2}\cdot(\sin(x))^{\frac{1}{2}-1}\cdot\cos(x)$

b) $\left(\sin\left(x^{\frac{1}{2}}\right)\right)' = \cos\left(x^{\frac{1}{2}}\right)\cdot\frac{1}{2}\cdot x^{\frac{1}{2}-1}$

c) $\left(\sqrt[7]{chx}\right)' = \left((ch(x))^{\frac{1}{7}}\right)' = \frac{1}{7}\cdot(ch(x))^{\frac{1}{7}-1}\cdot sh(x)$

10)a) $\left(\arcsin\frac{1}{x}\right)' = \arcsin'\left(\frac{1}{x}\right)\cdot\left(\frac{1}{x}\right)' = \frac{1}{\sqrt{1-\left(\frac{1}{x}\right)^2}}\cdot(-1)\cdot x^{-1-1}$,

b) $\left(\arccos(1-x^2)\right)' = \frac{-1}{\sqrt{1-(1-x^2)^2}}\cdot(0-2x)$,

c) $\left(\arcsin\left(2x\cdot\sqrt{1-x^2}\right)\right)' = \arcsin'\left(2x\cdot\sqrt{1-x^2}\right)\cdot\left(2x\cdot\sqrt{1-x^2}\right)' =$

$= \frac{1}{\sqrt{1-\left(2x\sqrt{1-x^2}\right)^2}}\cdot\left((2x)'\cdot\sqrt{1-x^2}+2x\cdot\left((1-x^2)^{\frac{1}{2}}\right)'\right) =$

$= \frac{1}{\sqrt{1-\left(2x\sqrt{1-x^2}\right)^2}}\cdot\left(2\cdot\sqrt{1-x^2}+2x\cdot\frac{1}{2}\cdot(1-x^2)^{\frac{1}{2}-1}\cdot(0-2x)\right)$,

11)a) $\left(\ln\frac{x}{2+3x}\right)' = \ln'\left(\frac{x}{2+3x}\right)\cdot\left(\frac{x}{2+3x}\right)' = \frac{1}{\frac{x}{2+3x}}\cdot\frac{x'\cdot(2+3x)-x\cdot(2+3x)'}{(2+3x)^2} =$

$= \frac{2+3x}{x}\cdot\frac{1\cdot(2+3x)-x\cdot 3}{(2+3x)^2}$,

b) $[\ln(\tan(x))]' = \ln'(\tan(x))\cdot\tan'(x) = \frac{1}{\tan(x)}\cdot\frac{1}{\cos^2(x)}$,

12)a) $\left(\ln\sqrt{\frac{5x}{2+3x}}\right)' = \ln'\left(\sqrt{\frac{5x}{2+3x}}\right)\cdot\left(\sqrt{\ }'\left(\frac{5x}{2+3x}\right)\right)\cdot\left(\frac{5x}{2+3x}\right)' =$

$= \frac{1}{\sqrt{\frac{5x}{2+3x}}}\cdot\frac{1}{2\sqrt{\frac{5x}{2+3x}}}\cdot\frac{(5x)'\cdot(2+3x)-5x\cdot(2+3x)'}{(2+3x)^2} =$

$= \frac{1}{\sqrt{\frac{5x}{2+3x}}}\cdot\frac{1}{2\sqrt{\frac{5x}{2+3x}}}\cdot\frac{5\cdot(2+3x)-5x\cdot 3x}{(2+3x)^2}$,

b) $\left(\log_2\sqrt{\frac{5x}{2+3x}}\right)' = \frac{1}{\sqrt{\frac{5x}{2+3x}}\cdot\ln(2)}\cdot\frac{1}{2\sqrt{\frac{5x}{2+3x}}}\cdot\frac{5\cdot(2+3x)-5x\cdot 3x}{(2+3x)^2}$,

13) $\left(\frac{x}{\sin x+\cos x}\right)' = \frac{x'\cdot(\sin x+\cos x)-x\cdot(\sin x+\cos x)'}{(\sin x+\cos x)^2} =$

$= \frac{1\cdot(\sin x+\cos x)-x\cdot(\cos x-\sin x)}{(\sin x+\cos x)^2}$,

14) $[(3x^2-2)\cdot e^2]' = (3x^2-2)'\cdot e^2 = (3\cdot 2\cdot x-0)\cdot e^2 = 6x\cdot e^2$,

15) $[(3x^2-2)\cdot e^{2x}]' = (3x^2-2)'\cdot e^{2x}+(3x^2-2)\cdot(e^{2x})' = (3\cdot 2\cdot x-0)\cdot e^{2x}+(3x^2-2)\cdot e^{2x}\cdot 2$

16) $[e^{2x}\cdot\cos 3x]' = (e^{2x})'\cdot\cos 3x+e^{2x}\cdot(\cos 3x)' = e^{2x}\cdot 2\cdot\cos 3x+e^{2x}\cdot(-\sin(3x))\cdot 3$,

17)a) $\left(\tan\left(\frac{1}{x^2}\right)\right)' = \tan'\left(\frac{1}{x^2}\right)\cdot\left(\frac{1}{x^2}\right)' = \frac{1}{\cos^2\left(\frac{1}{x^2}\right)}\cdot\frac{-2}{x^3}$,

b) $[x^2\cdot\tan(2x)]' = (x^2)'\cdot\tan(2x)+x^2\cdot\tan'(2x)\cdot(2x)' = 2x\cdot\tan(2x)+x^2\cdot\frac{1}{\cos^2(2x)}\cdot 2$,

18) $\left(\frac{1}{ch(x)}\right)' = \left(\frac{2}{e^x+e^{-x}}\right)' = 2\cdot\left[(e^x+e^{-x})^{-1}\right]' = 2\cdot(-1)\cdot(e^x+e^{-x})^{-2} = \frac{-2}{(e^x+e^{-x})^2}$,

19) $[\log_3(1-x^2)]' = \log_3'(1-x^2)\cdot(1-x^2)' = \frac{1}{(1-x^2)\cdot\ln(3)}\cdot(0-2x)$,

20) $\left(\ln\left(\frac{x^2-1}{x^2+1}\right)\right)' = \ln'\left(\frac{x^2-1}{x^2+1}\right)\cdot\left(\frac{x^2-1}{x^2+1}\right)' = \frac{x^2+1}{x^2-1}\cdot\frac{2x\cdot(x^2+1)-2x\cdot(x^2-1)}{(x^2+1)^2}$,

21) $\left(\sqrt{\frac{1+\cos x}{2}}\right)' = \sqrt{\ }'\left(\frac{1+\cos x}{2}\right)\cdot\left(\frac{1+\cos x}{2}\right)' = \frac{1}{2\sqrt{\frac{1+\cos x}{2}}}\cdot\frac{1}{2}\cdot(0-\sin(x))$,

22) $[x^3 \cdot (1-2x) \cdot (3x^2+4x)]' =$

$= (x^3)' \cdot (1-2x) \cdot (3x^2+4x) + x^3 \cdot (1-2x)' \cdot (3x^2+4x) + x^3 \cdot (1-2x) \cdot (3x^2+4x)' =$

$= 3x^2 \cdot (1-2x) \cdot (3x^2+4x) + x^3 \cdot (-2) \cdot (3x^2+4x) + x^3 \cdot (1-2x) \cdot (3 \cdot 2x + 4),$

23) $$\left(\frac{\sin(\cos(x^2))}{\exp_4\left(tg\left(\frac{1}{x^2}\right)\right)}\right)' = \frac{[\sin(\cos(x^2))]' \cdot \exp_4\left(tg\left(\frac{1}{x^2}\right)\right) - \sin(\cos(x^2)) \cdot \left[\exp_4\left(tg\left(\frac{1}{x^2}\right)\right)\right]'}{\left[\exp_4\left(tg\left(\frac{1}{x^2}\right)\right)\right]^2}$$

ahol $[\sin(\cos(x^2))]' = \sin'(\cos(x^2)) \cdot \cos'(x^2) \cdot (x^2)' = \cos(\cos(x^2)) \cdot (-\sin(x^2)) \cdot 2x$

és $\left[\exp_4\left(tg\left(\frac{1}{x^2}\right)\right)\right]' = \exp_4'\left(tg\left(\frac{1}{x^2}\right)\right) \cdot tg'\left(\frac{1}{x^2}\right) \cdot (x^2)' = \exp_4\left(tg\left(\frac{1}{x^2}\right)\right) \cdot \ln(4) \cdot \dfrac{1}{\cos^2\left(\frac{1}{x^2}\right)} \cdot 2x$.

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