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IDENTIFICATION OF PARAMETERS IN HEREDITARY SYSTEMS

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ABSTRACT

In this paper we study the numerical performance of a parameter identification technique, based on approximation by equations with piecewise constant arguments, on various classes of hereditary systems. The examples considered here include delay equations with state-dependent delays and neutral equations.

1. INTRODUCTION

Hereditary systems, i.e., systems with memory, appear as mathematical models in various applications in engineering. Many times the general form of the model is known or assumed, but the particular values of some parameters in the corresponding differential equation (such as coefficients, values of delays, the initial function, etc.) are not known, and have to be identified.

Consider e.g., the initial value problem (IVP) for the nonlinear delay system with state dependent delays

$$\dot{x}(t) = f\Big(t, x(t), x(t - \tau(t, x(t)))\Big), \qquad t \geq 0 \qquad (1.1)$$

with initial condition

$$x(t) = \varphi(t), \qquad t \in [-r, 0]. \qquad (1.2)$$

We assume that certain parameters, γ, in IVP (1.1)-(1.2) are not known explicitly, but some information is available on their values via measurements $(X_0, X_1, \ldots, X_l)$ of the solution, $x(t)$, at discrete time values $(t_0, t_1, \ldots, t_l)$. The goal is to find the parameter value, which minimizes the least squares fit-to-data criterion

$$J(\gamma) = \sum_{i=0}^{l} |x(t_i; \gamma) - X_i|^2, \qquad \gamma \in \Gamma,$$

i.e., which is the best-fit parameter for the measurements. (Denote this problem by $\mathcal{P}$). Problem $\mathcal{P}$ has been studied by many authors, for different classes of differential equations (see e.g. Banks and Kunish (1989) and the references therein), including delay equations (see e.g. Banks and Daniel (1982) and Murphy (1990)).

All the above cited papers use the same idea to find the solution of the optimization problem $\mathcal{P}$:

Step 1) First take finite dimensional approximations of the parameters, γ^N, (i.e., $\gamma^N \in \Gamma^N \subset \Gamma$, $\dim \Gamma^N < \infty$, $\gamma^N \to \gamma$ as $N \to \infty$).

Step 2) Consider a sequence of approximate initial value problems (IVP$_{M,N}$) corresponding to a discretization of IVP (1.1)-(1.2) for some fixed parameter $\gamma^N \in \Gamma^N$ with solutions $y^M(\cdot; \gamma^N)$ satisfying that $y^M(t, \gamma^N) \to x(t, \gamma)$ as $N, M \to \infty$, uniformly on compact time intervals.

Step 3) Define the least square minimization problems ($\mathcal{P}^{N,M}$) for each $N, M = 1, 2, \ldots$, i.e., find $\gamma^{N,M} \in \Gamma^N$, which minimizes the least squares fit-to-data criterion

$$J^{N,M}(\gamma^N) = \sum_{i=0}^{l} |y^M(t_i; \gamma^N) - X_i|^2, \qquad \gamma^N \in \Gamma^N.$$

Step 4) Assuming that the actual parameters belong to a compact subset of Γ, argue, that the sequence of solutions,

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$\gamma^{N,M}$ $(N, M = 1, 2, \ldots)$, of the finite dimensional minimization problems $\mathcal{P}^{N,M}$, has a convergent subsequence with limit $\bar{\gamma} \in \Gamma$.

Step 5) Show that $\bar{\gamma}$ is the solution of the minimization problem $\mathcal{P}$.

Note, that step 4) and 5) can be argued without using the particular approximation method of the initial value problem, using only compactness arguments and step 2) above (see e.g. in Murphy (1990)).

In Section 2 we define an Euler-type approximation scheme for a class of neutral equations which includes the delay equations with time- and state-dependent delays of form (1.1) as a special case. (This scheme was introduced by Győri (1991) for linear neutral equations with constant delays.) In Section 3 we present numerical examples illustrating the applicability of this identification method. Section 4 contains the conclusions.

2. NUMERICAL TECHNIQUE

Consider the vector neutral functional differential equation (NFDE)

$$\frac{d}{dt}\Big(x(t)+\sum_{i=1}^{m} q_i x(t-r_i)\Big) = f\Big(a, t, x(t), x(t-\tau(b,t,x(t)))\Big) \tag{2.1}$$

for $t \geq 0$, with initial condition

$$x(t) = \varphi(t), \qquad t \in [-r, 0]. \tag{2.2}$$

Here $a \in \mathbb{R}^s$ and $b \in \mathbb{R}^s$ on the righ hand side of equation (2.1) represent parameters in the equation and in the delay function, respectively, $q_i \in \mathbb{R}$ and $r_i \in [0, \infty)$ are constants $(i = 1, \ldots, m)$, $f : \mathbb{R}^s \times [0,\infty) \times \mathbb{R}^n \times \mathbb{R}^n \to \mathbb{R}^n$, $\tau : \mathbb{R}^s \times [0, \infty) \times \mathbb{R}^n \to [0, \infty)$, and $r \equiv \max\{r_1, \ldots, r_m, \sup_{t\geq 0, x\in\mathbb{R}^n} \tau(t, x)\}$, and the initial function $\varphi \in C \equiv C([-r, 0]; \mathbb{R}^n)$. Note that IVP (2.1)-(2.2) contains IVP (1.1)-(1.2) as a special case by taking $m = 1$ and $q_1 = 0$.

Our objective is to identify parameters a and b in the right hand side of the equation (2.1), the constants q_i and r_i of the left hand side of (2.1), and the initial function, φ. We define $\gamma \equiv (a, b, q_1, \ldots, q_m, r_1, \ldots, r_m, \varphi)$ for our parameter vector, and $\Gamma \equiv \mathbb{R}^s \times \mathbb{R}^s \times \mathbb{R}^m \times \mathbb{R}^m \times C$ for our parameter space.

Following the general method described in the Introduction, first we consider finite dimensional approximations $\gamma^N = (a^N, b^N, q_1^N, \ldots, r_1^N, \ldots, \varphi^N) \in \Gamma^N$ of parameter $\gamma \in \Gamma$. Of course we need to approximate only the last component, φ, of γ, since the other components are finite dimensional. (In the numerical examples we shall use linear spline approximations with equidistant mesh points. It is known (see e.g. Schultz, 1973) that by linear splines we can approximate piecewise smooth functions uniformly on compact time intervals.)

The second step is to define discretizations of IVP (2.1)-(2.2) with parameter γ^N. We use the natural generalization of the numerical scheme introduced by Győri (1991):

Let h be a positive number. Throughout this paper we shall use the notation $[t]_h \equiv [t/h]h$, where $[\cdot]$ is the greatest integer function. Elementary estimates give that $t - h < [t]_h \leq t$ and therefore $[t]_h \to t$ as $h \to 0$.

We associate the following NFDE with piecewise constant right-hand side to (2.1):

$$\begin{aligned} &\frac{d}{dt}\Big(y_h(t) + \sum_{i=1}^{m} q_i^N y_h(t - [r_i^N]_h)\Big) \\ &\quad = f\left(a^N, [t]_h, y_h([t]), y_h([t]_h - [\tau(b^N, [t]_h, y_h([t]_h))]_h\right). \end{aligned} \tag{2.3}$$

The subscript h of $y_h(t)$ emphasizes that $y_h(t)$ is the solution of (2.3) corresponding to the discretization parameter h. The associated initial condition to (2.3) is

$$y_h(t) = \varphi^N(t), \qquad t \in [-r, 0]. \tag{2.4}$$

By a solution of the initial value problem (2.3)-(2.4) we mean a function $y_h : [-r, T] \to \mathbb{R}^n$, which is defined on $[-r, 0]$ by (2.4) and satisfies the following properties on $[0, T]$:

(i) it is continuous on $[0, T]$,

(ii) the derivative $\frac{d}{dt}\left(y_h(t) + \sum_{i=1}^m q_i^N y_h(t - [r_i^N]_h)\right)$ exists at each point $t \in [0, \infty)$ with the possible exception of the points kh $(k = 0, 1, 2, \ldots)$ where finite one-sided derivatives exist,

(iii) y_h satisfies (2.3) on each interval $[kh, (k+1)h) \cap [0, T]$ for $k = 0, 1, 2, \ldots$.

Using the method of steps it can be verified that IVP (2.3)-(2.4) has a unique solution on $[0, \infty)$. Introduce the notation $z(k) \equiv y_h(kh)$. Then it is easy to see, using that the right hand side of (2.3) is constant on the intervals $[kh, (k + 1)h)$, that the sequence $z(k)$ satisfies

$$\begin{aligned} &z(k+1) \\ &\quad = z(k) + \sum_{i=1}^{m} q_i^N \Big(z(k - [r_i^N]_h) - z(k + 1 - [r_i^N]_h)\Big) \\ &\qquad + hf\Big(a^N, kh, z(k), z(k - [\tau(b^N, kh, z(k))]_h)\Big) \end{aligned}$$

for $k = 0, 1, \ldots$, and $z(k) = \varphi^N(kh)$ for negative integer k such that $-r \leq kh \leq 0$. Therefore computing $z(k)$ is a simple numerical task.

We conjecture the following result:

Theorem 2.1 *Assume that the functions f, τ and φ are locally Lipschitz-continuous in all of their variables. If $\gamma^N \to \gamma$ in some "appropriate" sense, then $y_h(t; \gamma^N) \to x(t; \gamma)$ uniformly on compact time intervals, as $h \to 0^+$, $N \to \infty$, where $x(t; \gamma)$ and $y_h(t; \gamma^N)$ are the solutions of IVP (2.1)-(2.2) and IVP (2.3)-(2.4) corresponding to parameter γ and γ^N, respectively.*

The proof of this theorem for state-dependent retarded delay equations and for a very similar approximating scheme can be found in Hartung (1995).

In practice we proceed as follows: We select "small enough" $h > 0$ and "large enough" N, and consider the least square criterion

$$J_h^N(\gamma^N) = \sum_{i=0}^{l} |y_h(t_i; \gamma^N) - X_i|^2, \qquad \gamma^N \in \Gamma^N,$$

then solve the (finite dimensional) minimization problem numerically, and use the solution of it as an approximation of the solution of the original identification problem. (Note that if the initial function is known then the parameter space is finite dimensional, and therefore there is no need for its discretization.)

3. CASE STUDIES

In this section we present some numerical examples to illustrate the identification method described in the Introduction and in Section 2. We note, that in Examples 3.1, 3.2, 3.4 and 3.5 we used the built in numerical minimization routine of Mathematica (which does not require the knowledge of the derivative of the minimizing function) for solving the finite dimensional minimization problems, i.e., for computing the minimum of the least square cost functions, and we also used Mathematica for evaluating the cost function for each required value of the parameter, i.e., for computing the solution of IVP (2.3)-(2.4). In Examples 3.3 and 3.6 (where we had higher dimensional optimization problems) we used our own nonlinear least square minimization code, based on secant method with Dennis-Gay-Welsch update, combined with trust region technique. See Section 10.3 in Dennis and Schnabel (1983) for detailed description of this method.

Example 3.1 Consider the scalar equation with constant delay

$$\dot{x}(t) = x(t-\tau), \qquad t \in [0,4], \tag{3.1}$$

with initial condition

$$x(t) = 1, \qquad t \in [-3,0]. \tag{3.2}$$

The solution of this IVP corresponding to $\tau = 1.0$ is

$$x(t;1) = \begin{cases} t+1, & t \in [0,1], \\ (t^2+3)/2, & t \in [1,2], \\ (t^3 - 3t^2 + 12t + 1)/6, & t \in [2,3], \\ (t^4 - 8t^3 + 42t^2 - 60t + 85)/24, & t \in [3,4]. \end{cases}$$

We used this formula to generate the "measured data" X_i corresponding to time values $t_i = 0.5i$, $(i = 0, \dots, 8)$. Our goal in this example is to identify the "true" delay, τ, using these measurements. Since the parameter is scalar, there is no need for discretizing the parameter space (Step 1 of the general method). To follow Step 2, let $h > 0$ and define the approximating IVP

$$\dot{y}_h(t) = y_h([t]_h - [\tau]_h), \qquad t \in [0,4], \tag{3.3}$$

with initial condition

$$y_h(t) = 1, \qquad t \in [-3,0]. \tag{3.4}$$

Following Step 3, consider the least squares function

$$J_h(\tau) = \sum_{i=0}^{8} (y_h(t_i;\tau) - X_i)^2, \qquad \tau \in [0.2, 3.0],$$

where $y_h(t;\tau)$ is the solution of (3.3)-(3.4). Here we made the apriori assumption that the parameter τ is in the compact set, $[0.2, 3]$. We present the numerical solution of these minimization problems in Table 3.1 for different h values using the initial guess $\tau = 2.5$ in each case. We print out the computed $\bar{\tau}$, which minimizes $J_h(\tau)$, and the corresponding value of the cost function.

This experiment shows that computed delay values approximate the true delay, $\tau = 1$, as h gets smaller.

Table 3.1

h	$\bar{\tau}$	$J_h(\bar{\tau})$
0.100	1.038789	0.0497927
0.050	1.012446	0.0128838
0.010	1.004631	0.0005232
0.005	1.001152	0.0001310
0.001	1.000847	0.0000052

Example 3.2 Consider the scalar delay equation with state-dependent delay

$$\dot{x}(t) = ax^2(t - b|x(t)|) + \sin(ct) + \sin^4(t - \sin^2(t)), \quad t \in [0,3], \tag{3.5}$$

with initial condition

$$x(t) = \sin^2(t), \qquad t \le 0, \tag{3.6}$$

where $\gamma = (a,b,c)$ are unknown parameters, but we assume that $\gamma \in \Gamma \equiv [-5,5] \times [0,5] \times [-5,5]$. It is easy to see that the solution of IVP (3.5)-(3.6) corresponding to parameter values $a = -1.0$, $b = 1.0$ and $c = 2.0$ is $x(t) = \sin^2 t$. We used this function to generate data $X_i = \sin^2 t_i$ corresponding to time values $t_i = 0.25i$, $(i = 0, \dots, 12)$.

The approximating equation corresponding to (3.5) is

$$\begin{aligned} \dot{y}_h(t) \quad = \quad & a y_h^2([t]_h - [b|y_h([t]_h)|]_h) + \sin(c[t]_h) \\ & + \sin^4\Big([t]_h - [\sin^2([t]_h)]_h\Big), \qquad t \in [0,3], \end{aligned}$$

and the function to be minimized is

$$J_h(\gamma) = \sum_{i=0}^{12} (y_h(t_i;\gamma) - X_i)^2, \qquad \gamma \in \Gamma.$$

Table 3.2 contains the numerical results. In this experiment we used initial guess 2.5 for all unknown parameters, a, b and c. In this example we also get a good approximation of the true parameter values.

Table 3.2

h	$\bar{a}$	$\bar{b}$	$\bar{c}$	$J_h(\bar{\gamma})$
0.100	-1.07566	1.06121	2.05960	0.00769018
0.050	-1.03784	1.03633	2.03225	0.00181179
0.010	-1.00759	1.00826	2.00687	0.00006892
0.005	-1.00373	1.00443	2.00355	0.00001710
0.001	-1.00074	1.00089	2.00071	0.00000068

Example 3.3 Consider the scalar equation

$$\dot{x}(t) = x\left(t - 1 - \frac{1}{t+1}\right), \qquad t \in [0,2], \tag{3.7}$$

with initial condition

$$x(t) = \varphi(t), \qquad t \in [-2,0]. \tag{3.8}$$

It is easy to check that the solution of IVP (3.7)-(3.8) with initial function

$$\varphi(t) = \begin{cases} \frac{2}{3}(t+2), & -2 \le t \le -0.5, \\ 1, & -0.5 \le t \le 0 \end{cases} \tag{3.9}$$

is

$$x(t) = \begin{cases} 1 + \frac{2}{3}t + \frac{t^3}{3} - \frac{2}{3}\log(t+1), & t \in [0,1], \\ 1 - \frac{2}{3}\log 2 + t, & t \in [1,2]. \end{cases}$$

We generated measurements X_i corresponding to time values $t_i = 0.1i$, $(i = 0, \ldots, 20)$ by using this function.

The approximate equation corresponding to (3.7) is

$$\dot{y}_h(t) = y_h\left([t]_h - \left[1 + \frac{1}{[t]_h + 1}\right]_h\right), \qquad t \in [0,2].$$

First we approximate the unknown initial function on $[-2,0]$ by linear spline functions with three node points at -2, -1 and 0, with corresponding values a_1, a_2 and a_3 at the node points. We assume that the parameter values satisfy $a_i \in [-4,4]$, $i = 1,2,3$, i.e., $\gamma \equiv (a_1, a_2, a_3) \in \Gamma \equiv [-4,4]^3$. Then the parameter space is three dimensional. The corresponding cost function is:

$$J_h(\gamma) = \sum_{i=0}^{20} (y_h(t_i;\gamma) - X_i)^2, \qquad \gamma \in \Gamma. \tag{3.10}$$

We present the numerical solution of this problem in Table 3.3 for several h values.

Next we consider 5 and 7 dimensional linear spline approximations of the initial function with node points at $T_i = -2 + (i-1)/2$, $i = 1, \ldots, 5$ and $T_i = -2 + (i-1)/3$, $i = 1, \ldots, 7$, respectively, and with the corresponding values a_i ($i = 1, 2, \ldots, 5$ and $i = 1, 2, \ldots, 7$, respectively) at

Table 3.3

h	$\bar{a}_1$	$\bar{a}_2$	$\bar{a}_3$	$J_h(\bar{\gamma})$
0.100	-0.312237	0.858357	1.032257	0.0069884
0.050	-0.376395	0.854938	1.032996	0.0069854
0.010	-0.419767	0.854892	1.032343	0.0068774
0.005	-0.423922	0.854539	1.032307	0.0068753
0.001	-0.426665	0.854214	1.032277	0.0068822

these points. Let $\gamma \equiv (a_1, \ldots, a_5)$ and $\Gamma \equiv [-4,4]^5$, and $\gamma \equiv (a_1, \ldots, a_7)$ and $\Gamma \equiv [-4,4]^7$, respectively. Then the parameter space is five and seven dimensional, and the minimizing function, (3.10), is of five and seven variables. The corresponding numerical results are shown in Table 3.4 and Table 3.5. In Figure 3.1 we plotted the true initial function, defined by (3.9) (solid line), and the computed approximate initial functions with three, five and seven node points (dotted linear splines) corresponding to $h = 0.001$.

Table 3.4

h	$\bar{a}_1$	$\bar{a}_2$	$\bar{a}_3$
0.100	0.098056	0.150230	0.761572
0.050	0.088556	0.143898	0.742056
0.010	0.081379	0.140601	0.732469
0.005	0.080761	0.140047	0.731171
0.001	0.080713	0.139400	0.730222

h	$\bar{a}_4$	$\bar{a}_5$	$J_h(\bar{\gamma})$
0.100	1.068640	0.992574	0.0022000
0.050	1.075129	0.992861	0.0020954
0.010	1.074786	0.992916	0.0021417
0.005	1.074835	0.992928	0.0021403
0.001	1.074952	0.992918	0.0021432

Table 3.5

h	$\bar{a}_1$	$\bar{a}_2$	$\bar{a}_3$	$\bar{a}_4$
0.100	0.055464	0.117036	0.371743	0.637042
0.050	0.030480	0.114776	0.358293	0.611259
0.010	0.007429	0.112583	0.348555	0.596303
0.005	0.006507	0.111256	0.347952	0.595390
0.001	0.005875	0.110454	0.347464	0.594431

h	$\bar{a}_5$	$\bar{a}_6$	$\bar{a}_7$	$J_h(\bar{\gamma})$
0.100	1.140893	0.950127	0.997607	0.0013675
0.050	1.177600	0.954204	0.998046	0.0012264
0.010	1.170907	0.957232	0.998201	0.0012996
0.005	1.170391	0.957714	0.998191	0.0013049
0.001	1.170289	0.957939	0.998181	0.0013094

Example 3.4 Our next example is the scalar NFDE

$$\frac{d}{dt}\Big(x(t) + qx(t-1)\Big) = x(t-2), \quad t \ge 0, \tag{3.11}$$

$$x(t) = \varphi(t), \qquad t \in [-2,0], \tag{3.12}$$

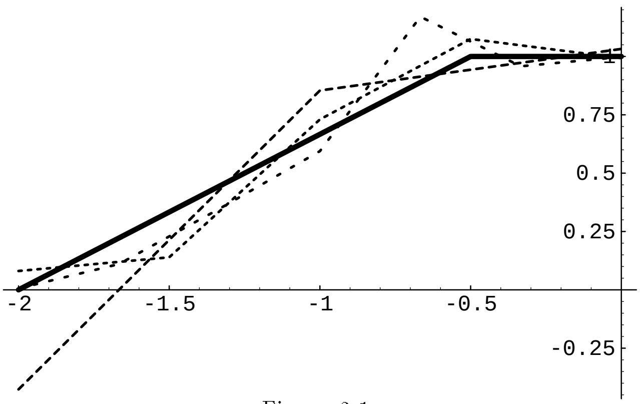

Figure 3.1

where the initial function is

$$\varphi(t) \equiv \begin{cases} t+2, & t \in [-2,-1], \\ t^2, & t \in [-1,0]. \end{cases} \tag{3.13}$$

It is easy to obtain by the method of steps that the solution of IVP (3.11)-(3.12) corresponding to $q = -0.5$ is

$$x(t) = \begin{cases} t^2 - t & t \in [0,1], \\ \frac{1}{3}t^3 - \frac{3}{2}t^2 + \frac{5}{2}t - \frac{4}{3} & t \in [1,2], \\ \frac{1}{2}t^3 - \frac{15}{4}t^2 + \frac{37}{4}t - \frac{43}{6} & t \in [2,3]. \end{cases} \tag{3.14}$$

The approximate IVP is

$$\begin{aligned} \frac{d}{dt}\Big(y_h(t) + qy(t-[1]_h)\Big) &= y_h([t]_h - [2]_h), \quad t \in [0,3], \\ y_h(t) &= \varphi(t), \quad t \in [-2,0]. \end{aligned}$$

We minimize

$$J_h(q) = \sum_{i=0}^{12} (y_h(t_i;q) - X_i)^2, \qquad q \in [-3.0, 3.0],$$

where we use the formula of the true solution to generate data X_i corresponding to time values $t_i = 0.25i$, $i = 0,\ldots,12$. The numerical results of the minimization problems, corresponding to the initial guess $q = 0$, are shown in Table 3.6.

Table 3.6

h	$\bar{q}$	$J_h(\bar{q})$
0.100	-0.486359	0.0033643
0.050	-0.492758	0.0008282
0.010	-0.498492	0.0000317
0.005	-0.499242	0.0000079
0.001	-0.499848	0.0000003

Example 3.5 Consider the scalar NFDE

$$\frac{d}{dt}\Big(x(t) - 0.5x(t-r)\Big) = x(t-2), \qquad t \in [0,3]$$

with initial condition (3.12)-(3.13). Then, as we have seen in the previous example, the solution corresponding to $r = 1$ is given by (3.14). The objective of this example is the identification of the delay r, where we make the assumption that $r \in [0.2, 3]$. We use the measurements of the previous example. The cost function is

$$J_h(r) = \sum_{i=0}^{12} (y_h(t_i;r) - X_i)^2, \qquad r \in [0.2, 3.0].$$

The numerical results, corresponding to initial guess $r = 2$, are presented in Table 3.7.

Table 3.7

h	$\bar{r}$	$J_h(\bar{r})$
0.100	1.06389	0.008292296
0.050	1.02878	0.002227256
0.010	1.01193	0.001047141
0.005	1.00212	0.000023315
0.001	1.00054	0.000000936

Example 3.6 Consider again the scalar NFDE

$$\begin{aligned} \frac{d}{dt}\Big(x(t) - 0.5x(t-1)\Big) &= x(t-2), \qquad t \in [0,2], \\ x(t) &= \varphi(t), \qquad t \in [-2,0]. \end{aligned}$$

The solution corresponding to initial function (3.13) is given by (3.14). In this example we would like to identify the initial function, using the measurements taken at the points $t_i = 0.1i$, $i = 0,\ldots,30$ and the formula (3.14). Since the initial function is infinite dimensional, first we approximate it using linear spline functions with equidistant mesh points. In the fist case consider a 3 dimensional approximation, i.e., a linear spline with 3 mesh points at -2, -1 and at 0 with corresponding function values a_1, a_2 and a_3. We assume that $\gamma \equiv (a_1,a_2,a_3) \in \Gamma \equiv [-4,4]^3$, and we minimize the cost function

$$J_h(\gamma) = \sum_{i=0}^{30} (y_h(t_i;\gamma) - X_i)^2, \qquad \gamma \in \Gamma.$$

Table 3.8 presents our numerical findings, using initial guesses $a_i = 0$ ($i = 1,2,3$). Table 3.9 and Table 3.10 contain the respective numerical results for 5 and 7 dimensional spline approximation, using constant zero function as the initial guess for φ. We show the true initial function (solid line) and the identified initial functions (dotted graphs) using 3, 5 and 7 dimensional spline approximation and discretization parameter $h = 0.001$ in Figure 3.2. In this exmple, despite the low ($N = 3$, 5 and 7) dimensional approximation of the initial function, we get a "good" identification of the initial function.

Table 3.8

h	$\bar{a}_1$	$\bar{a}_2$	$\bar{a}_3$	$J_h(\bar{\gamma})$
0.100	0.135754	0.803422	-0.092011	0.0609659
0.050	0.085954	0.842106	-0.083065	0.0547063
0.010	0.039925	0.874002	-0.075508	0.0494167
0.005	0.033756	0.878037	-0.074540	0.0487403
0.001	0.028753	0.881273	-0.073762	0.0481969

Table 3.9

h	$\bar{a}_1$	$\bar{a}_2$	$\bar{a}_3$
0.100	0.037568	0.533903	0.899094
0.050	0.020140	0.521371	0.936173
0.010	0.001626	0.509293	0.965196
0.005	-0.001013	0.507636	0.968778
0.001	-0.003178	0.506285	0.971635

h	$\bar{a}_4$	$\bar{a}_5$	$J_h(\bar{\gamma})$
0.100	0.137601	-0.014958	0.0027388
0.050	0.168314	-0.014970	0.0023605
0.010	0.195577	-0.014960	0.0021681
0.005	0.199150	-0.014954	0.0021505
0.001	0.202034	-0.014948	0.0021373

Table 3.10

h	$\bar{a}_1$	$\bar{a}_2$	$\bar{a}_3$	$\bar{a}_4$
0.100	-0.049855	0.396062	0.674139	0.896522
0.050	-0.055801	0.386530	0.663527	0.945686
0.010	-0.063387	0.377568	0.653336	0.982823
0.005	-0.064632	0.376323	0.651967	0.987291
0.001	-0.065686	0.375301	0.650856	0.990834

h	$\bar{a}_5$	$\bar{a}_6$	$\bar{a}_7$	$J_h(\bar{\gamma})$
0.100	0.364778	0.034412	-0.002997	0.0006052
0.050	0.396703	0.058223	-0.002624	0.0003475
0.010	0.423355	0.078563	-0.002712	0.0002502
0.005	0.426764	0.081188	-0.002744	0.0002449
0.001	0.429503	0.083302	-0.002772	0.0002418

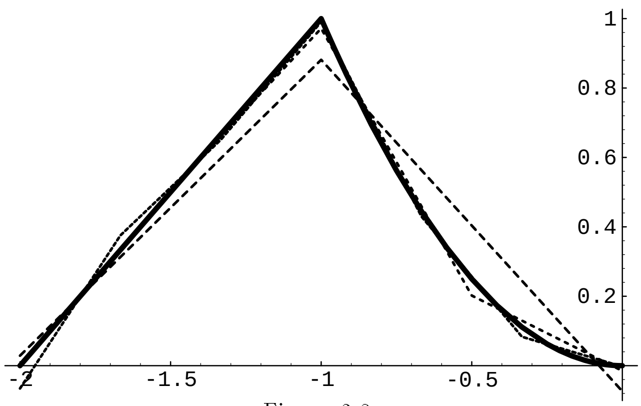

Figure 3.2

4. CONCLUSIONS

In this paper we experimented with a numerical method for parameter identification for various classes of hereditary systems. Although our results are preliminary in nature, the indication is that the method is applicable even to certain neutral functional differential equations. We intend to continue this study in forthcoming publications.

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