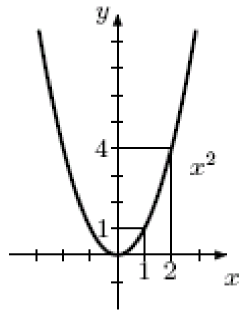
1. ***<u>Hatványfüggvények</u>*** $f(x)=x^a$ $(a\in\mathrm{R})$

1.1. $a = n\in N^+$

a)

$$f(x) = x^n \qquad (n \in \mathbb{N}^+)$$

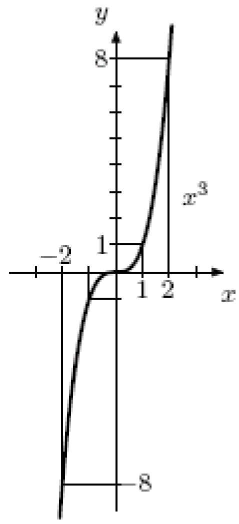
n páros (Pl.: x^2, x^4, x^6 stb.)



$D_f = \mathbb{R}$
$R_f = [0, \infty)$
ZH = $\{0\}$
páros
nem monoton (szig. mon. csökkenő $(-\infty, 0]$-on,
szig. mon. növekvő $[0, \infty)$-on)

b)

n páratlan (Pl.: x^3, x^5, x^7 stb.)



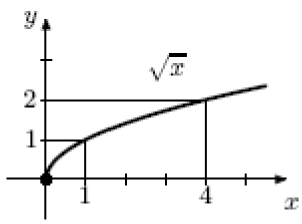
$D_f = \mathbb{R}$
$R_f = \mathbb{R}$
ZH = $\{0\}$
páratlan
szig mon. növő

1.2. $a = 1/n$

a)

$$f(x) = \sqrt[n]{x} \qquad (n \in \{2, 3, 4, \dots\})$$

n páros (Pl.: $\sqrt{x}$, $\sqrt[4]{x}$, $\sqrt[6]{x}$ stb.)

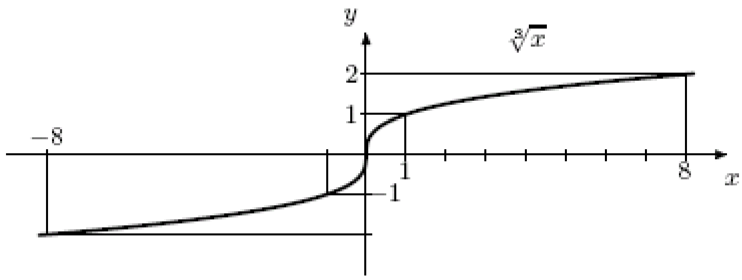


$D_f = [0, \infty)$
$R_f = [0, \infty)$
ZH = $\{0\}$

szig. mon. növő

b)

n páratlan (Pl.: $\sqrt[3]{x}$, $\sqrt[5]{x}$, $\sqrt[7]{x}$ stb.)

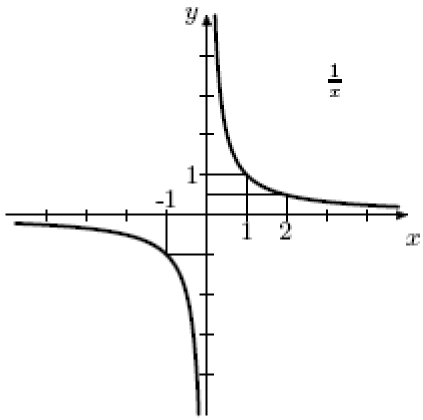


$D_f = \mathbb{R}$
$R_f = \mathbb{R}$
ZH = $\{0\}$
páratlan
szig. mon. növő

1.3. *a<0* , *a*∈Z

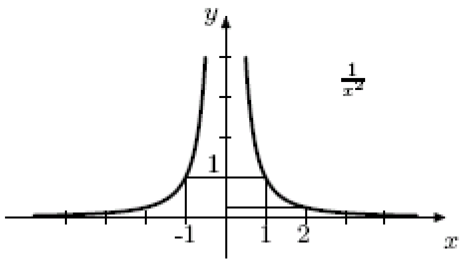
$f(x) = \frac{1}{x^n}$ $(n \in \mathbb{N}^+)$

a) n páratlan (Pl.: $\frac{1}{x}, \frac{1}{x^3}$ stb.)



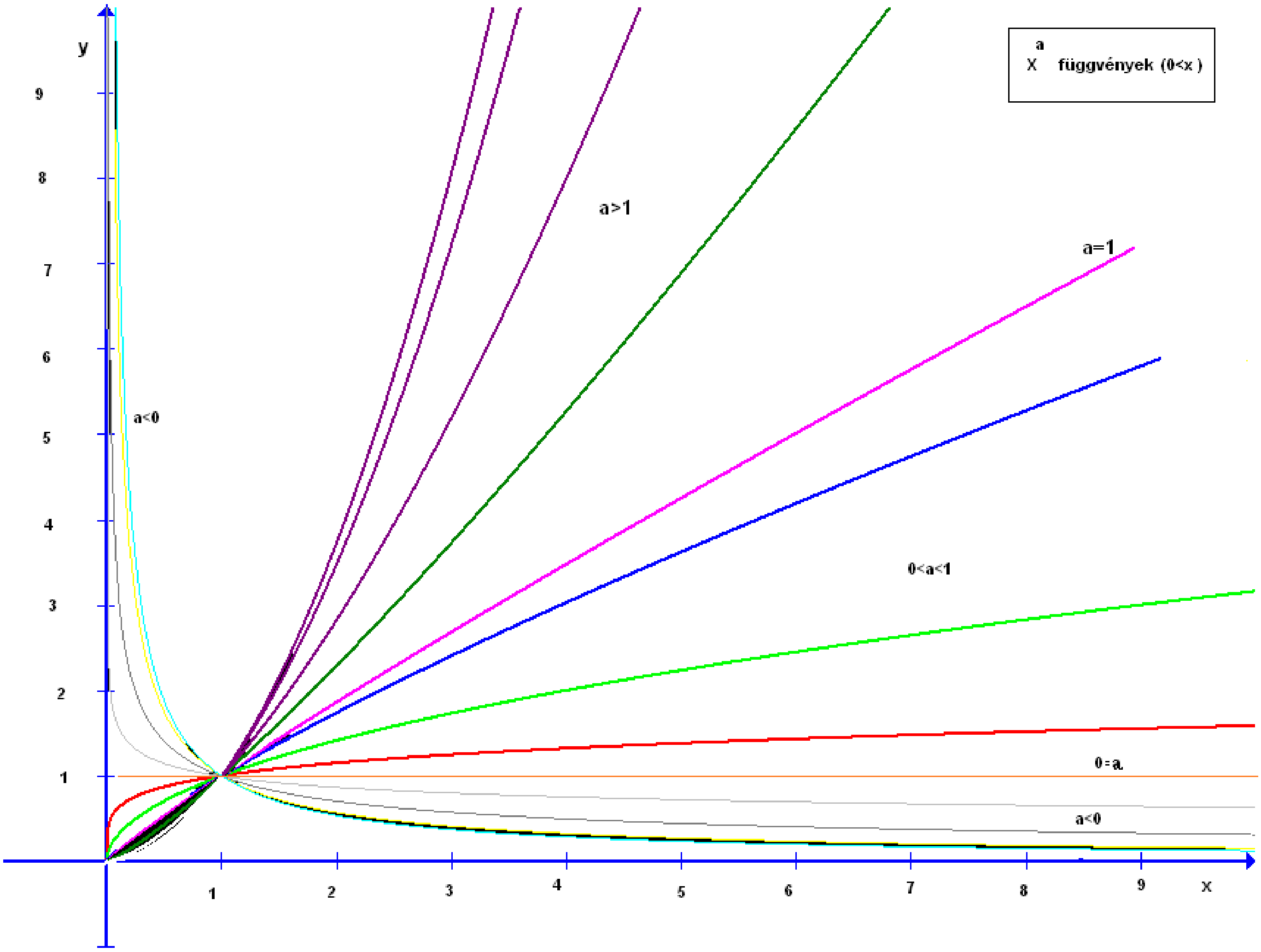
$D_f = \mathbb{R} \setminus \{0\}$
$R_f = \mathbb{R} \setminus \{0\}$
$ZH = \emptyset$
páratlan
nem monoton (szig. mon. csökkenő a $(-\infty, 0)$ és a $(0, \infty)$-okon)

b) n páros (Pl.: $\frac{1}{x^2}, \frac{1}{x^4}$ stb.)



$D_f = \mathbb{R} \setminus \{0\}$
$R_f = \mathbb{R}^+$
$ZH = \emptyset$
páros
nem monoton (szig. mon. növekvő a $(-\infty, 0)$-on, szig. mon. csökkenő a $(0, \infty)$-on)

1.4. *Összehasonlítás: a*∈R, 0<x

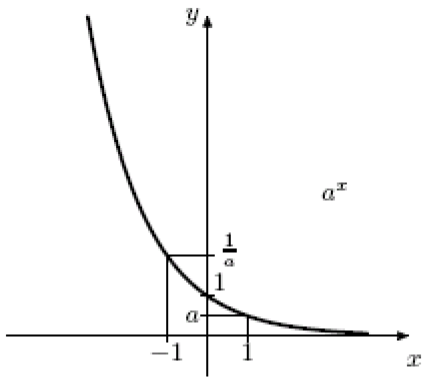


2. *Exponenciális és logaritmus függvények*

2.1.

a) $f(x) = a^x$ $(a \in \mathbb{R}^+ \setminus \{1\})$

$a \in (0,1)$ (Pl.: $\left(\frac{1}{2}\right)^x, \left(\frac{1}{10}\right)^x, \left(\frac{1}{e}\right)^x$ stb.)

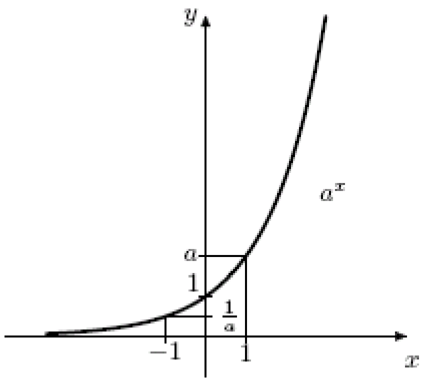


$D_f = \mathbb{R}$
$R_f = (0, \infty)$
$ZH = \emptyset$

szig. mon. csökkenő

b)

$a \in (1,\infty)$ (Pl.: $2^x, 10^x, e^x$ stb.)

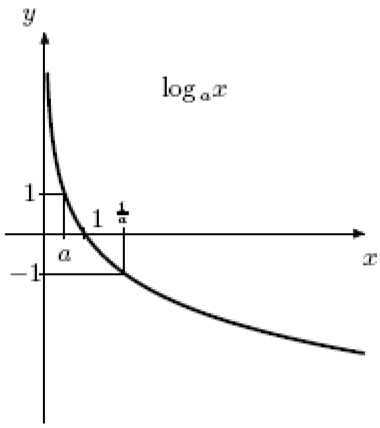


$D_f = \mathbb{R}$
$R_f = (0, \infty)$
$ZH = \emptyset$

szig. mon. növő

2.2. $f(x) = \log_a x$ $(a \in \mathbb{R}^+ \setminus \{1\})$

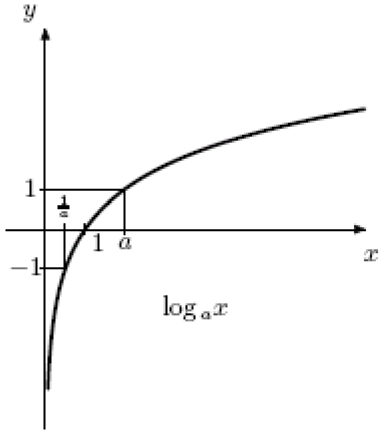
a) $a \in (0,1)$ (Pl.: $\log_{\frac{1}{2}} x, \log_{\frac{1}{10}} x, \log_{\frac{1}{e}} x$ stb.)



$D_f = \mathbb{R}^+$
$R_f = \mathbb{R}$
$ZH = \{1\}$

szig. mon. csökkenő

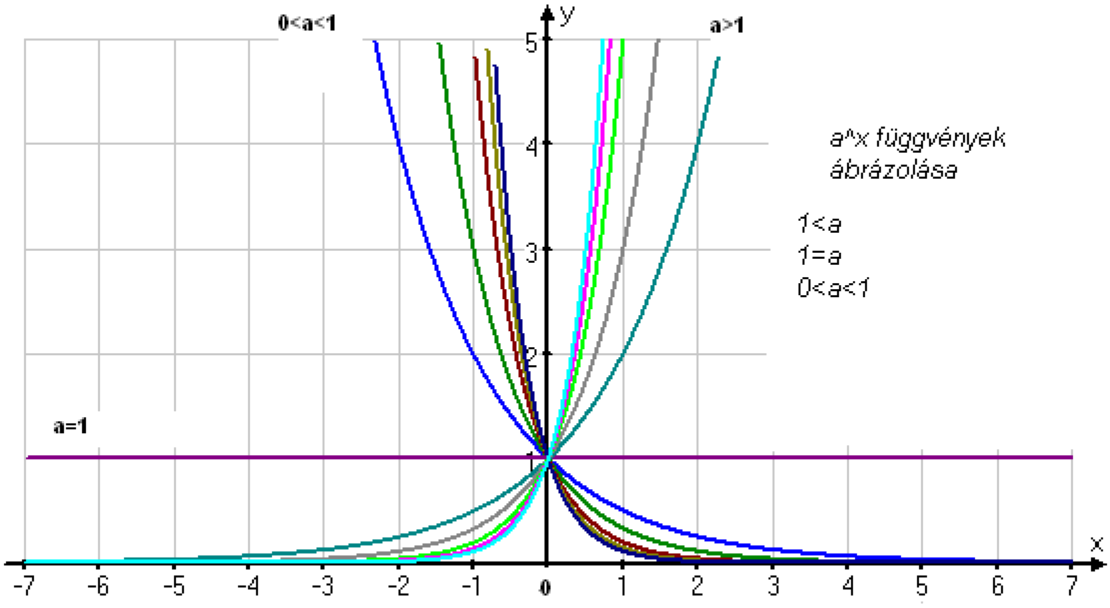
b) $a \in (1,\infty)$ (Pl.: $\log_2 x, \lg x, \ln x$ stb.)

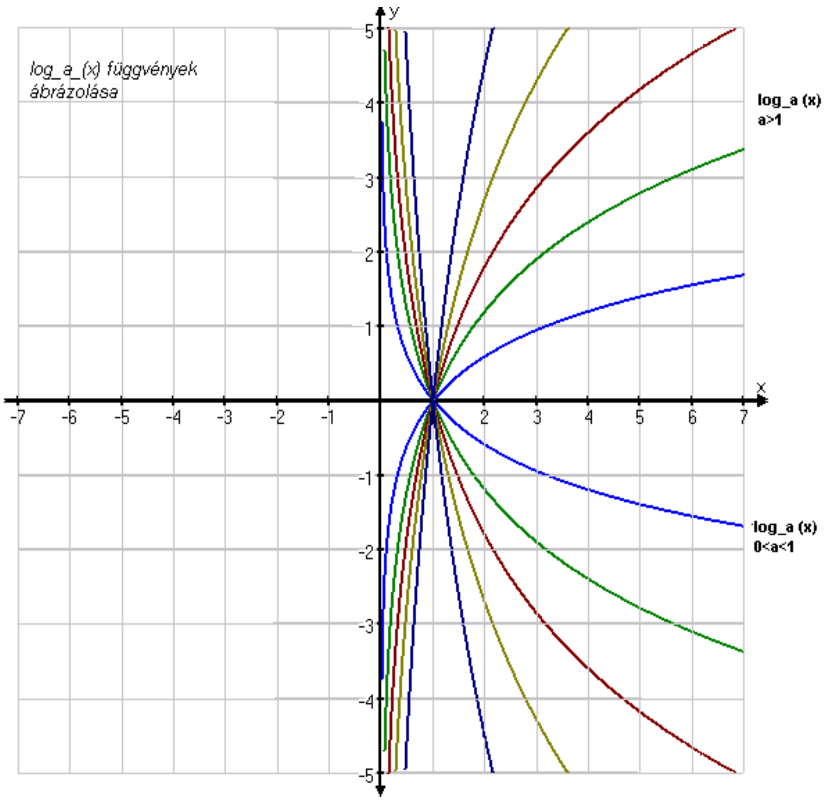


$D_f = \mathbb{R}^+$
$R_f = \mathbb{R}$
$ZH = \{1\}$

szig. mon. növő

2.3. *Összefoglaló ábrák*

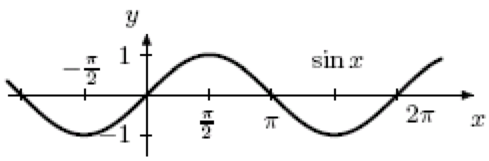




3.a) ***Trigonometrikus függvények***

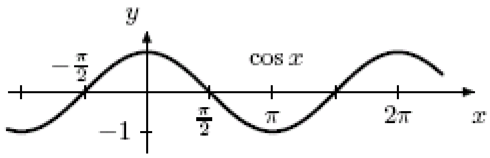
1. $f(x) = \sin x$

$D_f = \mathbb{R}$
$R_f = [-1,1]$
$ZH = \{k\pi \mid k \in \mathbb{Z}\}$
páratlan, periodikus (p=2π)
nem monoton (szig. mon. növő a $(-\frac{\pi}{2} + 2k\pi, \frac{\pi}{2} + 2k\pi)$-okon $(k \in \mathbb{Z})$,
szig. mon. csökkenő a $(\frac{\pi}{2} + 2k\pi, \frac{3\pi}{2} + 2k\pi)$-okon $(k \in \mathbb{Z})$)



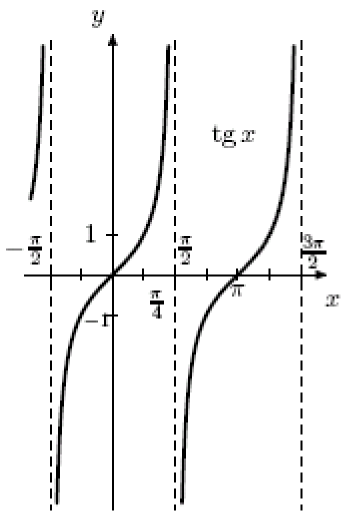
2. $f(x) = \cos x$

$D_f = \mathbb{R}$
$R_f = [-1,1]$
$ZH = \{\frac{\pi}{2} + k\pi \mid k \in \mathbb{Z}\}$
páros, periodikus (p=2π)
nem monoton (szig. mon. növő a $(-\pi + 2k\pi, 2k\pi)$-okon $(k \in \mathbb{Z})$,
szig. mon. csökkenő a $(2k\pi, \pi + 2k\pi)$-okon $(k \in \mathbb{Z})$)



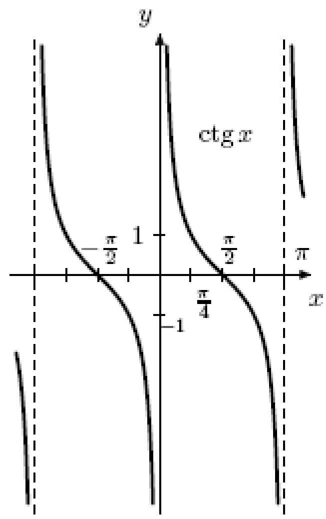
3. $f(x) = \operatorname{tg} x$

$D_f = \mathbb{R} \setminus \{\frac{\pi}{2} + k\pi \mid k \in \mathbb{Z}\}$
$R_f = \mathbb{R}$
$ZH = \{k\pi \mid k \in \mathbb{Z}\}$
páratlan, periodikus (p=π)
nem monoton (szig. mon. növő a $(-\frac{\pi}{2} + k\pi, \frac{\pi}{2} + k\pi)$-okon $(k \in \mathbb{Z})$)



4. $f(x) = \operatorname{ctg} x$

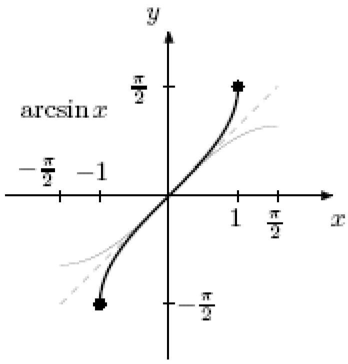
$D_f = \mathbb{R} \setminus \{k\pi \mid k \in \mathbb{Z}\}$
$R_f = \mathbb{R}$
$ZH = \{\frac{\pi}{2} + k\pi \mid k \in \mathbb{Z}\}$
páratlan, periodikus (p=π)
nem monoton (szig. mon. csökkenő a $(k\pi, \pi + k\pi)$-okon $(k \in \mathbb{Z})$)



3.b) ***Trigonometrikus függvények inverzei (arcus- fv.)***

1. $f(x) = \arcsin x$

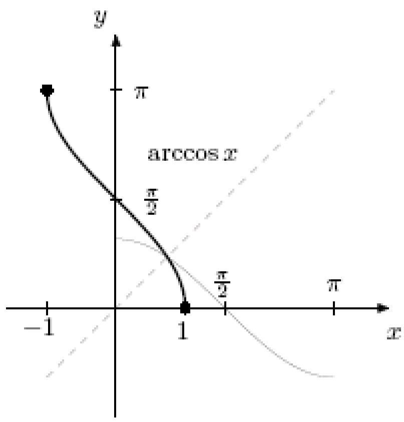
$D_f = [-1, 1]$
$R_f = [-\frac{\pi}{2}, \frac{\pi}{2}]$
$ZH = \{0\}$
páratlan
szig. mon. növő



2. $f(x) = \arccos x$

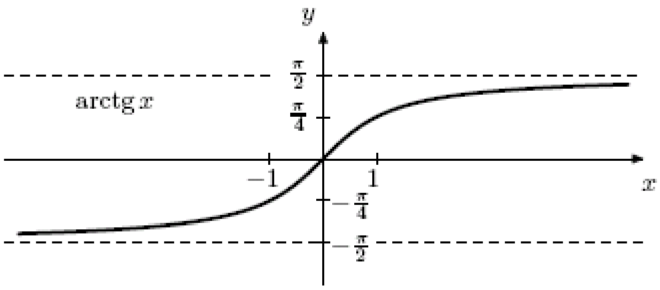
$D_f = [-1, 1]$
$R_f = [0, \pi]$
$ZH = \{1\}$

szig. mon. csökkenő



3. $f(x) = \operatorname{arctg} x$

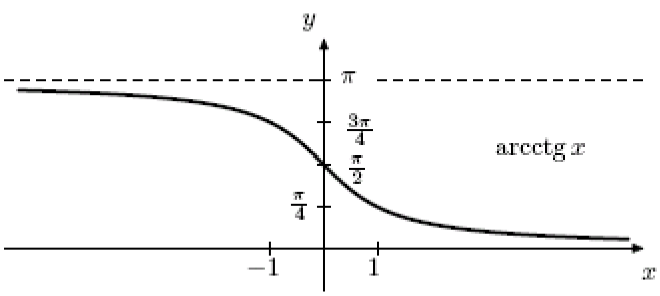
$D_f = \mathbb{R}$
$R_f = (-\frac{\pi}{2}, \frac{\pi}{2})$
$ZH = \{0\}$
páratlan
szig. mon. növő



4. $f(x) = \operatorname{arcctg} x$

$D_f = \mathbb{R}$
$R_f = (0, \pi)$
$ZH = \emptyset$

szig. mon. csökkenő



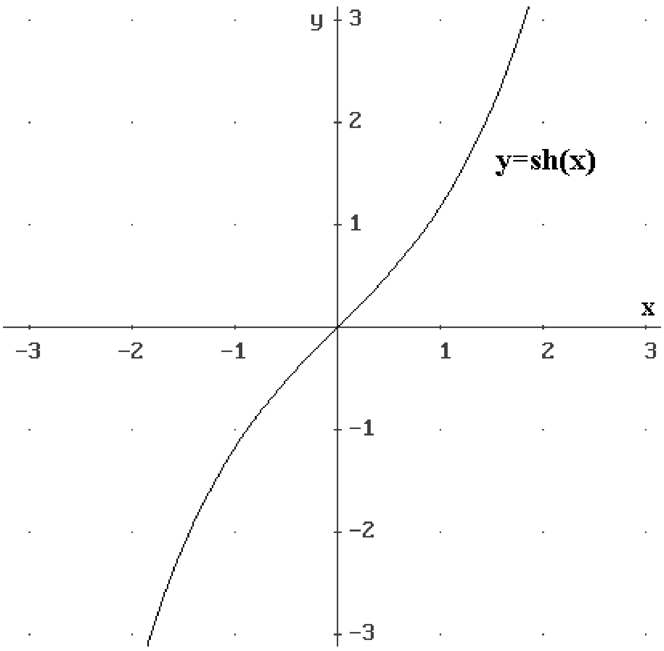
4. *Hiperbolikus függvények és inverzeik*

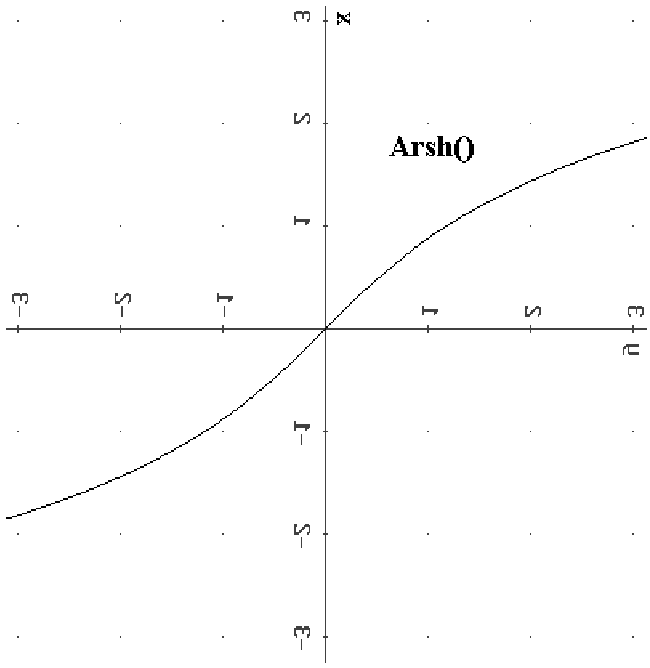
4.1. sh(x):= $\frac{e^x - e^{-x}}{2}$ (*sinus hyperbolicus*)

Dom(sh)=R,   Im(sh)=R,

páratlan fv., szig. mon. nő   =>   invertálható: (*Area sinus hyp.*)

$\lim_{-\infty} \text{sh}(x) = -\infty$, $\lim_{+\infty} \text{sh}(x) = +\infty$.   **$\text{sh}^{-1}$(y) = Arsh(y) =** $\ln(y + \sqrt{y^2 + 1})$



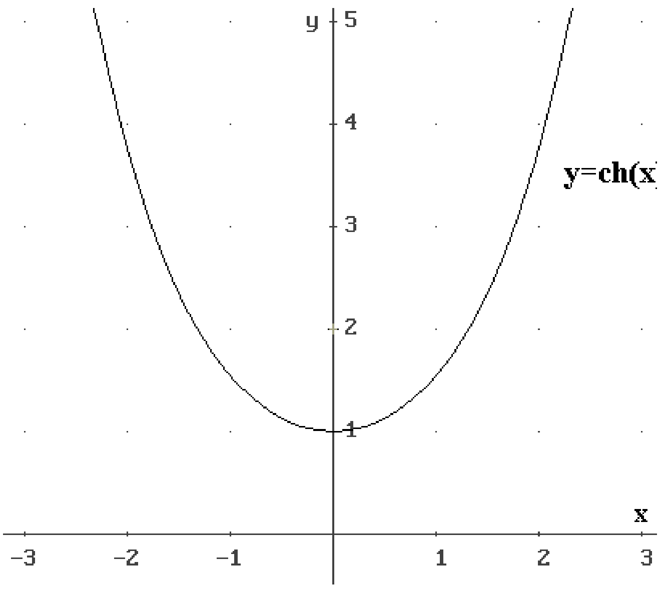


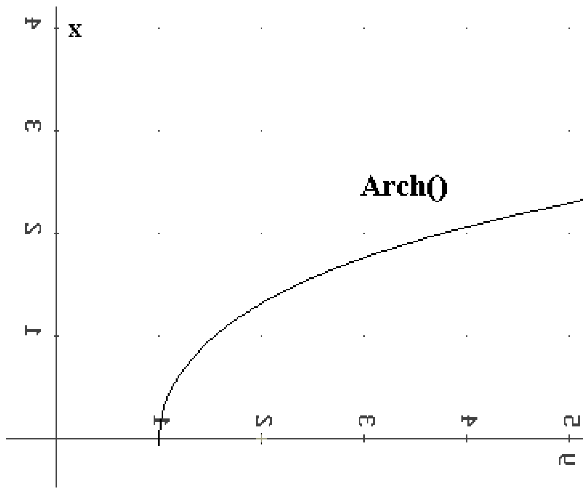
4.2. ch(x):= $\frac{e^x + e^{-x}}{2}$ (*cosinus hyperbolicus*)

Dom(ch)=R,   Im(ch)=R,

páros fv., x>0 esetén szig. mon. nő   =>   invertálható: (*Area cosinus hyp.*)

$\lim_{-\infty} \text{ch}(x) = \lim_{+\infty} \text{ch}(x) = +\infty$.   **$\text{ch}^{-1}$(y) = Arch(y) =** $\ln(y + \sqrt{y^2 - 1})$



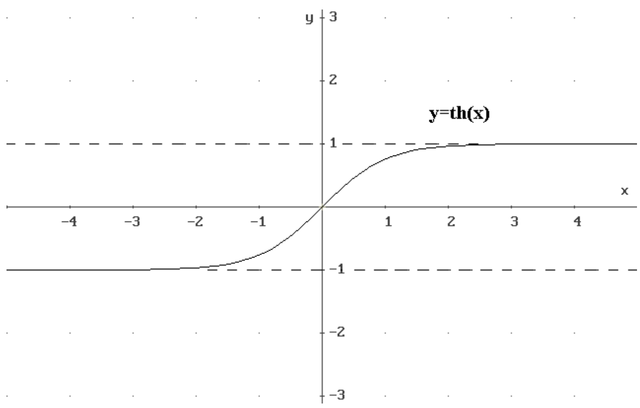


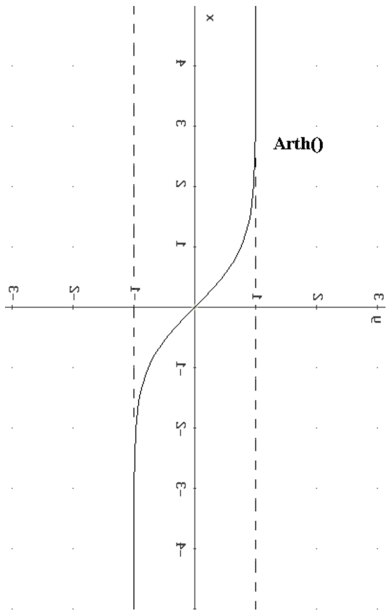
4.3. th(x):= $\frac{\text{sh}(x)}{\text{ch}(x)} = \frac{e^x - e^{-x}}{e^x + e^{-x}}$ (*tangens hyperbolicus*)

Dom(th)=R, Im(th)=(0,1),

páratlan fv., szig. mon. nő => invertálható: (*Area tangens hyp.*)

$\lim_{-\infty} \text{th}(x) = -1$, $\lim_{+\infty} \text{th}(x) = +1$. **th^{-1}(y) = Arth(y) =** $\frac{1}{2}\ln\left(\frac{1+y}{1-y}\right)$ (|y|<1)



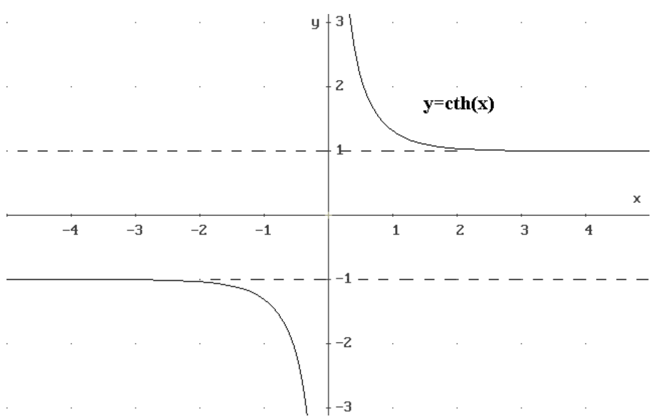


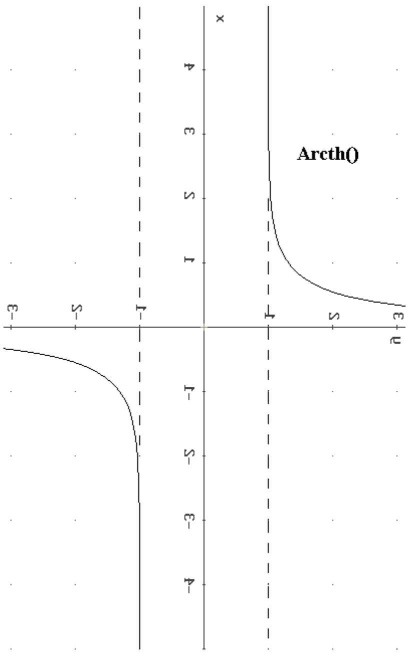
4.4. cth(x):= $\frac{1}{\text{th}(x)} = \frac{e^x + e^{-x}}{e^x - e^{-x}}$ (*cotangens hyperbolicus*)

Dom(cth)=R\{0}, Im(cth)=R\[0,1],

páratlan fv., két ága szig. mon. csökken => invertálható: (*Area cotangens hyp.*)

$\lim_{-\infty} \text{cth}(x) = -1$, $\lim_{+\infty} \text{cth}(x) = +1$. **cth^{-1}(y) = Arcth(y)=** $\frac{1}{2}\ln\left(\frac{1+y}{1-y}\right)$ (|y|>1)





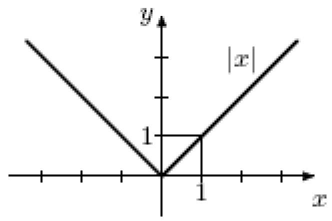
5. *Egyéb (nem elemi) függvények*

1. $f(x) = |x|$

Az abszolútérték definíciója:

$$|x| := \begin{cases} x, & \text{ha} \quad x \geq 0 \\ -x, & \text{ha} \quad x < 0 \end{cases}$$

$D_f = \mathbb{R}$
$R_f = [0, \infty)$
ZH $= \{0\}$
páros
nem monoton (szig. mon. csökkenő $(-\infty, 0]$-on,
szig. mon. növekvő $[0, \infty)$-on)

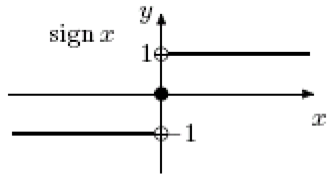


2. $f(x) = \operatorname{sign} x$

A sign definíciója:

$$\operatorname{sign} x := \begin{cases} \frac{x}{|x|}, & \text{ha} \quad x \neq 0 \\ 0, & \text{ha} \quad x = 0 \end{cases}$$

$D_f = \mathbb{R}$
$R_f = \{-1, 0, 1\}$
ZH $= \{0\}$
páratlan
monoton növő

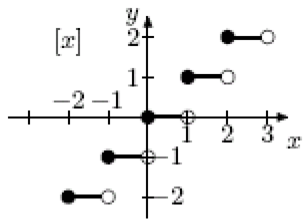


3. $f(x) = [x]$

Az egészrész definíciója: $[x] := \max\{ m \mid m \in \mathbb{Z}, m \leq x\}$

$D_f = \mathbb{R}$
$R_f = \mathbb{Z}$
ZH $= [0, 1)$

monoton növő



4. $f(x) = \{x\}$

A törtrész definíciója: $\{x\} := x - [x]$

$D_f = \mathbb{R}$
$R_f = [0, 1)$
ZH $= \mathbb{Z}$
periodikus (p=1)
nem monoton (szig. mon. növekvő az $[m, m+1)$
intervallumokon $(m \in \mathbb{Z})$)